Example paper: Black-Box Optimization Benchmarking Template for the Comparison of More than Two Algorithms on the Noiseless Testbed

Draft version [*]

BBOBies

ABSTRACT

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. RESULTS

Results from experiments according to [**?**] on the benchmark functions given in [**?**, **?**] are presented in Figures 1, 2 and 3 and in Tables 1 and 2. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_\mathrm{t} = f_\mathrm{opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [**?**, **?**]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t (10^{-8} as in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

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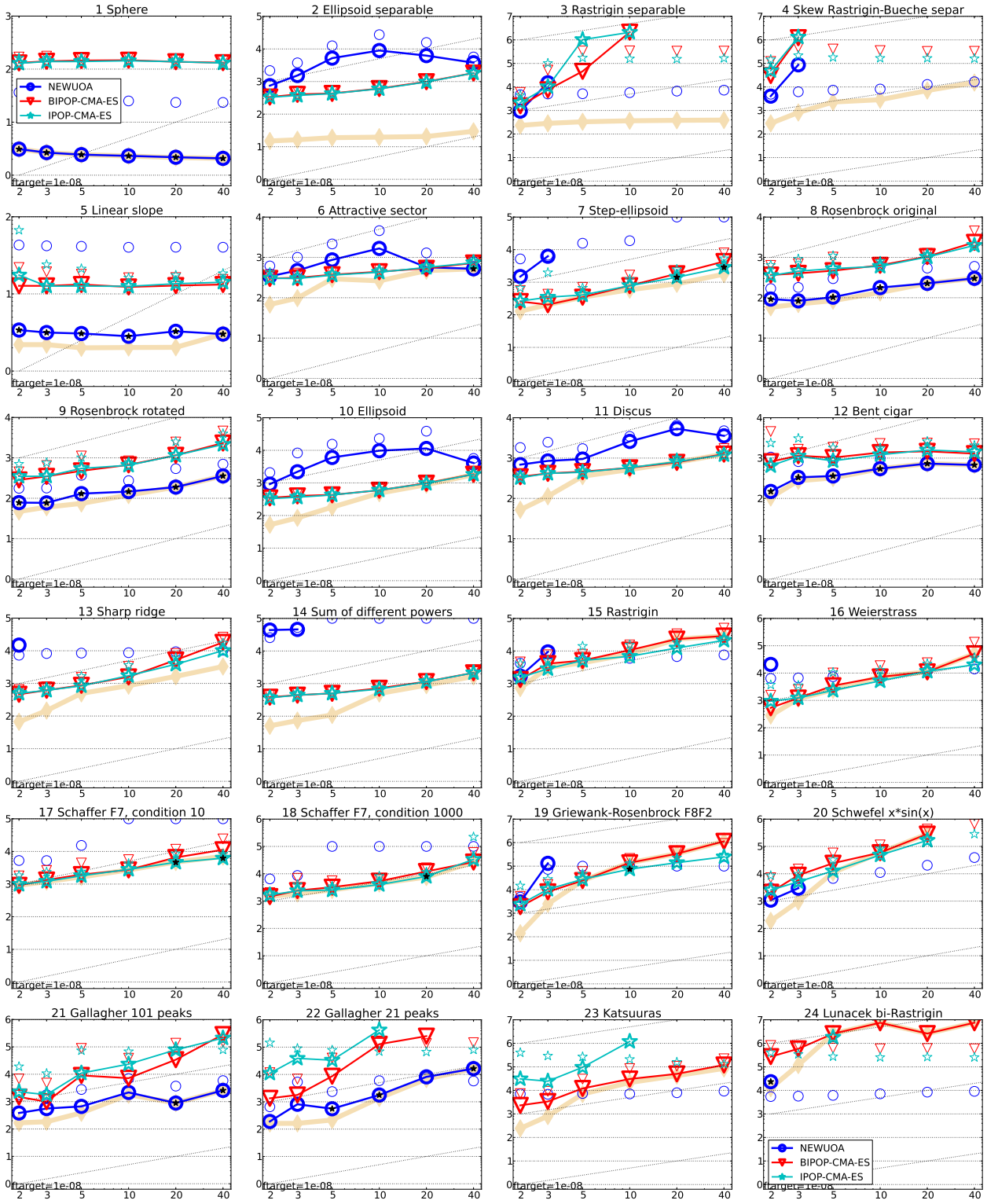

Figure 1: Expected running time (ERT in number of f-evaluations) divided by dimension for target function value 10^{-8} as $\log_{10}$ values versus dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○: NEWUOA, ▽: BIPOP-CMA-ES, ⋆: IPOP-CMA-ES.

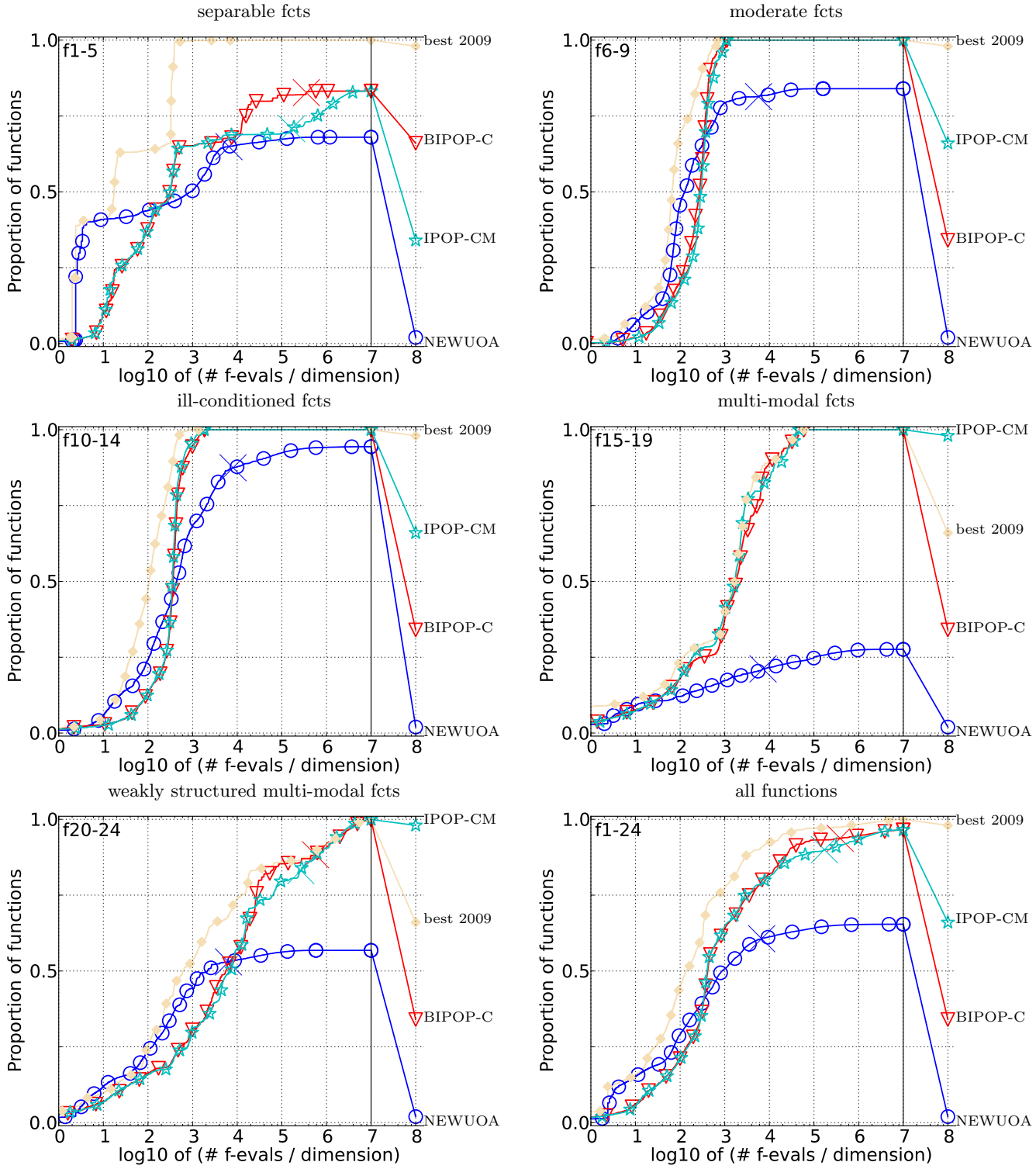

Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

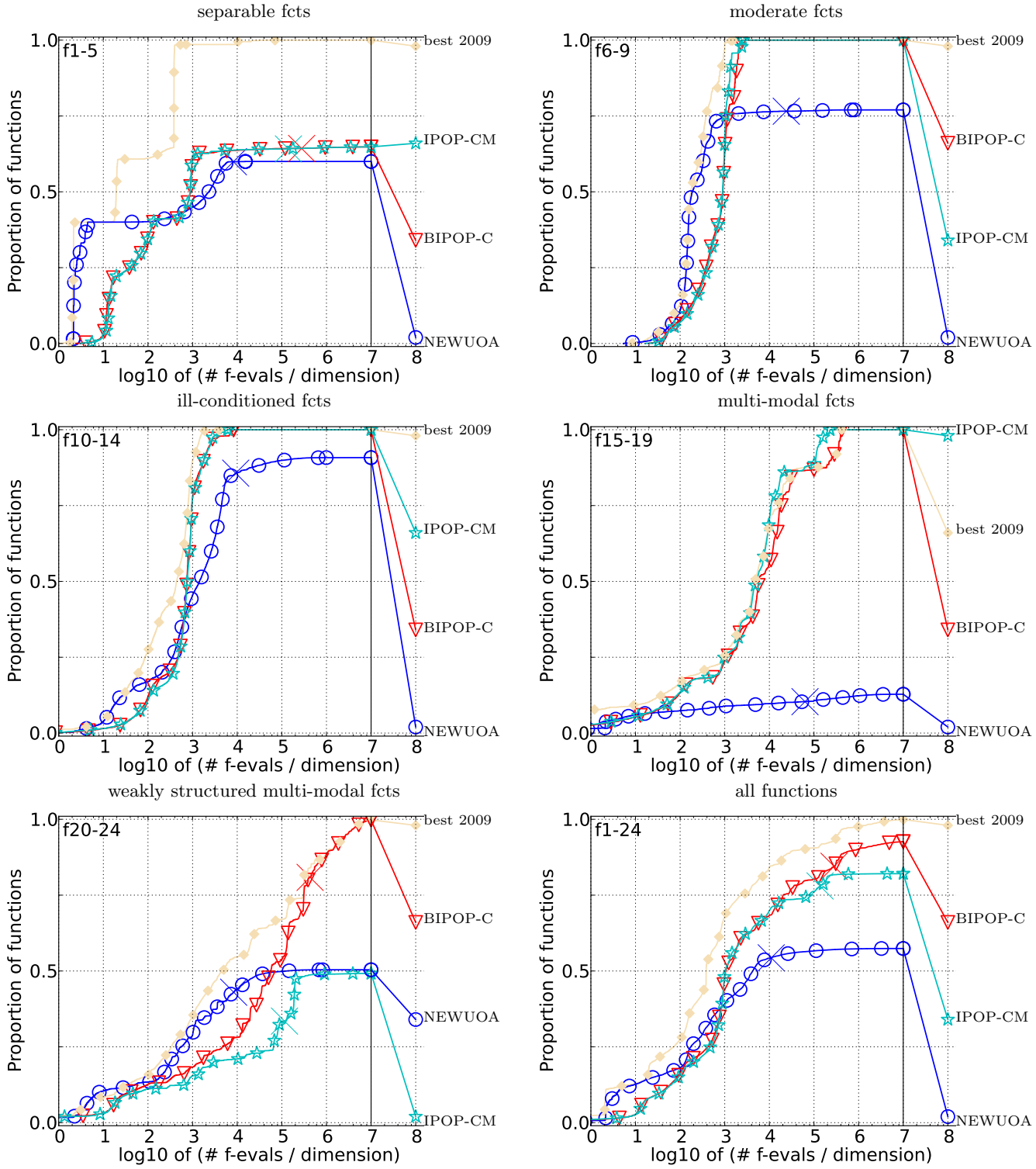

Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	15/15
NEWUOA	**1.1**(0)	**1**(0)$^{\star 4}$	**1**(0)$^{\star 4}$	**1**(0)$^{\star 4}$	**1**(0)$^{\star 4}$	**1**(0)$^{\star 4}$	15/15
BIPOP-C	3.2(2)	9.0(4)	15(4)	27(5)	40(4)	53(6)	15/15
IPOP-CM	2.5(2)	8.0(3)	14(3)	27(3)	39(3)	51(3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f2	83	87	88	90	92	94	15/15
NEWUOA	**5.7**(4)$^{\star 2}$	22(16)	45(30)	85(32)	129(33)	166(54)	15/15
BIPOP-C	13(4)	**16**(3)	**18**(2)	20(2)	21(2)	22(2)	15/15
IPOP-CM	14(4)	16(4)	18(2)	**19**(2)	**21**(2)	**22**(2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f3	716	1622	1637	1646	1650	1654	15/15
NEWUOA	6.1(8)	229(246)	∞	∞	∞	∞ *3e4*	0/15
BIPOP-C	**1.4**(1)	**16**(17)	**139**(107)	**139**(107)	**139**(107)	**140**(107)	15/15
IPOP-CM	2.2(4)	70(138)	3130(3633)	3113(3795)	3106(4028)	3099(3614)	2/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4	809	1633	1688	1817	1886	1903	15/15
NEWUOA	27(25)	**305**(338)	∞	∞	∞	∞ *3e4*	0/15
BIPOP-C	2.7(3)	∞	∞	∞	∞	∞ *2e6*	0/15
IPOP-CM	**2.0**(3)	∞	∞	∞	∞	∞ *9e5*	0/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f5	10	10	10	10	10	10	15/15
NEWUOA	**1.3**(0.1)$^{\star 4}$	**1.5**(0.2)$^{\star 4}$	**1.5**(0.2)$^{\star 4}$	**1.5**(0.2)$^{\star 4}$	**1.5**(0.2)$^{\star 4}$	**1.5**(0.2)$^{\star 4}$	15/15
BIPOP-C	4.5(2)	6.5(3)	6.6(2)	6.6(2)	6.6(2)	6.6(2)	15/15
IPOP-CM	4.6(2)	6.0(3)	6.3(3)	6.3(3)	6.3(3)	6.3(3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f6	114	214	281	580	1038	1332	15/15
NEWUOA	**1.7**(2)	2.4(1)	3.6(2)	3.3(2)	2.7(1)	2.9(1)	15/15
BIPOP-C	2.3(1)	**2.1**(0.6)	2.2(0.6)	**1.7**(0.2)	1.3(0.3)	1.3(0.2)	15/15
IPOP-CM	2.5(0.9)	2.1(0.6)	**2.2**(0.4)	1.7(0.2)	**1.3**(0.2)	**1.2**(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f7	24	324	1171	1572	1572	1597	15/15
NEWUOA	10(15)	13(18)	60(59)	∞	∞	∞ *3e4*	0/15
BIPOP-C	5.0(5)	**1.5**(1)	**1**(1)	**1**(0.9)	**1**(0.9)	**1**(0.9)	15/15
IPOP-CM	**4.4**(3)	1.7(1)	1.2(0.9)	1.2(0.6)	1.2(0.6)	1.2(0.6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f8	73	273	336	391	410	422	15/15
NEWUOA	**1**(0.9)$^{\star 3}$	**1.1**(0.8)$^{\star 2}$	**1.2**(0.5)$^{\star 3}$	**1.2**(0.4)$^{\star 4}$	**1.2**(0.4)$^{\star 4}$	**1.2**(0.4)$^{\star 4}$	15/15
BIPOP-C	3.2(1)	3.7(2)	4.5(2)	4.8(2)	5.1(2)	5.4(2)	15/15
IPOP-CM	3.5(2)	4.8(5)	5.3(4)	5.6(4)	5.8(3)	6.1(3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f9	35	127	214	300	335	369	15/15
NEWUOA	**1.8**(0.7)$^{\star 4}$	**3.6**(3)	**2.5**(2)$^{\star 2}$	**1.9**(1)$^{\star 3}$	**1.9**(1)$^{\star 3}$	**1.7**(0.9)$^{\star 4}$	15/15
BIPOP-C	5.8(2)	8.7(3)	7.2(2)	6.4(2)	6.3(1)	6.2(1)	15/15
IPOP-CM	6.0(2)	11(10)	8.7(6)	7.5(5)	7.3(4)	7.2(4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f10	349	500	574	626	829	880	15/15
NEWUOA	**3.1**(3)	5.5(4)	8.1(7)	14(9)	16(8)	21(8)	15/15
BIPOP-C	3.5(0.8)	**2.9**(0.4)	**2.7**(0.4)	2.8(0.2)	2.3(0.2)	2.4(0.1)	15/15
IPOP-CM	3.6(1.0)	2.9(0.5)	2.7(0.3)	**2.8**(0.3)	**2.3**(0.2)	**2.3**(0.2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f11	143	202	763	1177	1467	1673	15/15
NEWUOA	**3.5**(2)$^{\star 3}$	**4.7**(2)*	**1.8**(0.6)	1.8(0.4)	2.0(0.3)	2.2(0.4)	15/15
BIPOP-C	8.4(3)	7.2(2)	2.2(0.3)	1.6(0.2)	1.4(0.1)	1.3(0.1)	15/15
IPOP-CM	8.6(2)	7.3(1)	2.1(0.2)	**1.6**(0.1)	**1.4**(0.1)	**1.3**(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f12	108	268	371	461	1303	1494	15/15
NEWUOA	**3.5**(3)	**2.6**(3)	**2.5**(2)*	**2.6**(2)*	**1.1**(1)*	**1.1**(1.0)	15/15
BIPOP-C	11(12)	7.4(8)	7.4(6)	7.7(5)	3.3(2)	3.3(2)	15/15
IPOP-CM	9.4(7)	6.1(5)	6.2(4)	6.3(4)	2.8(2)	2.8(2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f13	132	195	250	1310	1752	2255	15/15
NEWUOA	**3.1**(3)	9.3(12)	35(36)	54(54)	335(373)	∞ *4e4*	0/15
BIPOP-C	3.9(3)	5.4(3)	5.9(3)	1.6(0.3)	**1.5**(0.2)	1.7(0.8)	15/15
IPOP-CM	3.1(1)	**5.0**(2)	**5.3**(3)	**1.4**(0.6)	1.6(0.3)	**1.6**(0.6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f14	10	41	58	139	251	476	15/15
NEWUOA	1.7(0.6)	**1**(0.4)$^{\star 3}$	**1**(0.3)$^{\star 4}$	**1.2**(0.3)$^{\star 4}$	5.5(2)	2525(2961)	0/15
BIPOP-C	**1.1**(1.0)	2.8(1)	3.7(0.9)	4.6(0.7)	**5.4**(0.5)	4.5(0.3)	15/15
IPOP-CM	2.2(3)	2.9(2)	3.8(1)	4.7(1)	5.4(0.7)	**4.4**(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f15	511	9310	19369	20073	20769	21359	14/15
NEWUOA	5.8(6)	41(45)	∞	∞	∞	∞ *3e4*	0/15
BIPOP-C	**1.6**(2)	1.5(1)	1.2(0.7)	1.2(0.7)	1.2(0.7)	1.2(0.7)	15/15
IPOP-CM	2.3(3)	**1.3**(0.8)	**1.2**(1)	**1.2**(1)	**1.2**(1)	**1.2**(1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f16	120	612	2662	10449	11644	12095	15/15
NEWUOA	**2.1**(2)	29(23)	∞	∞	∞	∞ *4e4*	0/15
BIPOP-C	3.0(3)	3.6(3)	2.6(1)	1.3(2)	1.4(2)	1.4(2)	15/15
IPOP-CM	2.5(2)	**2.3**(3)	**1.7**(1)	**0.96**(0.9)	**0.94**(0.7)	**0.95**(0.7)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f17	5.2	215	899	3669	6351	7934	15/15
NEWUOA	**2.3**(2)	40(47)	617(689)	∞	∞	∞ *3e4*	0/15
BIPOP-C	3.4(3)	**1**(0.2)	1(2)	1(0.7)	1(0.5)	1.2(0.5)	15/15
IPOP-CM	4.8(6)	1.1(0.5)	**0.97**(2)	**0.77**(1)	**0.81**(0.7)	**1.0**(0.4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f18	103	378	3968	9280	10905	12469	15/15
NEWUOA	31(28)	1351(1817)	∞	∞	∞	∞ *9e4*	0/15
BIPOP-C	**1**(0.7)	3.4(5)	1(1)	**1**(0.3)	1.2(0.7)	1.3(0.6)	15/15
IPOP-CM	1.2(0.5)	**2.7**(6)	**0.87**(1)	1.0(0.4)	**1.0**(0.3)	**0.99**(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f19	1	1	242	1.2e5	1.2e5	1.2e5	15/15
NEWUOA	**14**(2)	2.7e4(2e4)	1415(1927)	∞	∞	∞ *5e5*	0/15
BIPOP-C	20(16)	2801(5070)	161(175)	**1**(0.7)	**1**(0.7)	**1**(0.7)	15/15
IPOP-CM	21(25)	**1720**(1760)	**125**(94)	1.1(0.7)	1.1(0.7)	1.1(0.7)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f20	16	851	38111	54470	54861	55313	14/15
NEWUOA	**1**(0.2)$^{\star 2}$	**3.3**(4)	∞	∞	∞	∞ *3e4*	0/15
BIPOP-C	3.3(3)	8.2(10)	2.8(1)	2.1(0.8)	2.2(0.8)	2.2(0.8)	15/15
IPOP-CM	3.9(2)	11(11)	**1.4**(0.9)	**1.1**(0.7)	**1.1**(0.7)	**1.1**(0.7)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f21	41	1157	1674	1705	1729	1757	14/15
NEWUOA	**1.1**(0.6)	**2.2**(3)	**1.8**(2)	**1.8**(2)	**1.8**(2)	**1.9**(2)	15/15
BIPOP-C	2.3(2)	14(9)	24(35)	25(36)	25(36)	25(36)	15/15
IPOP-CM	6.3(17)	5.6(6)	30(34)	31(38)	31(41)	31(40)	14/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f22	71	386	938	1008	1040	1068	14/15
NEWUOA	**2.1**(3)	**2.1**(2)	**2.0**(3)*	**2.1**(3)*	**2.3**(3)	**2.4**(3)	15/15
BIPOP-C	6.9(11)	20(14)	45(94)	42(88)	41(85)	40(83)	15/15
IPOP-CM	12(26)	48(77)	166(219)	161(220)	158(211)	155(196)	11/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f23	3.0	518	14249	31654	33030	34256	15/15
NEWUOA	6.2(4)	**2.4**(2)	7.1(8)	∞	∞	∞ *3e4*	0/15
BIPOP-C	**1.7**(2)	13(15)	**3.7**(4)	**1.8**(2)	**1.8**(2)	**1.8**(2)	15/15
IPOP-CM	2.2(2)	26(32)	33(47)	15(21)	14(20)	14(20)	11/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f24	1622	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
NEWUOA	2.9(2)	2.1(2)	∞	∞	∞	∞ *3e4*	0/15
BIPOP-C	**2.1**(2)	**1.6**(3)	**1**(1.0)	1(1.0)	1(1)	1(1)	3/15
IPOP-CM	2.9(3)	18(22)	1.4(2)	**0.94**(1)	**0.70**(0.8)	**0.70**(0.8)	2/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 5. The inter-80%tile range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\text{opt}} + 10^{-8}$. Best results are printed in bold.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	15/15
NEWUOA	**1.0**(0.0)$^{\star 4}$	**1.0**(0.0)$^{\star 4}$	**1.0**(0.0)$^{\star 4}$	**1.0**(0.0)$^{\star 4}$	**1.0**(0.0)$^{\star 4}$	**1.0**(0.0)$^{\star 4}$	15/15
BIPOP-C	7.9(2)	14(3)	20(2)	33(4)	45(3)	57(3)	15/15
IPOP-CM	8.0(1)	14(2)	20(2)	33(1)	46(2)	58(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f2	385	386	387	390	391	393	15/15
NEWUOA	**18**(8)$^{\star 3}$	42(21)	71(36)	125(43)	174(51)	219(67)	15/15
BIPOP-C	35(7)	**40**(4)	44(4)	47(2)	48(2)	50(2)	15/15
IPOP-CM	35(4)	41(4)	**43**(3)	**45**(3)	**47**(2)	**48**(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f3	5066	7626	7635	7643	7646	7651	15/15
NEWUOA	∞	∞	∞	∞	∞	∞ *1e5*	0/15
BIPOP-C	**12**(7)	∞	∞	∞	∞	∞ *6e6*	0/15
IPOP-CM	13(6)	∞	∞	∞	∞	∞ *3e6*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4	4722	7628	7666	7700	7758	1.4e5	9/15
NEWUOA	∞	∞	∞	∞	∞	∞ *2e5*	0/15
BIPOP-C	∞	∞	∞	∞	∞	∞ *6e6*	0/15
IPOP-CM	∞	∞	∞	∞	∞	∞ *3e6*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f5	41	41	41	41	41	41	15/15
NEWUOA	**1.2**(0.1)$^{\star 4}$	**1.5**(0.4)$^{\star 4}$	**1.6**(0.5)$^{\star 4}$	**1.6**(0.5)$^{\star 4}$	**1.6**(0.5)$^{\star 4}$	**1.6**(0.5)$^{\star 4}$	15/15
BIPOP-C	5.1(0.8)	6.2(1)	6.3(1)	6.3(1)	6.3(1)	6.3(1)	15/15
IPOP-CM	5.8(1)	6.5(1)	6.7(1.0)	6.7(1.0)	6.7(1.0)	6.7(1.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f6	1296	2343	3413	5220	6728	8409	15/15
NEWUOA	**1**(0.3)$^{\star 2}$	**1**(0.4)	**1**(0.5)	**1.1**(0.5)	1.3(0.8)	1.3(0.7)	15/15
BIPOP-C	1.5(0.4)	1.3(0.2)	1.2(0.2)	1.1(0.2)	**1.2**(0.1)	**1.2**(0.1)	15/15
IPOP-CM	1.7(0.3)	1.3(0.1)	1.2(0.1)	1.2(0.1)	1.2(0.1)	1.2(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f7	1351	4274	9503	16524	16524	16969	15/15
NEWUOA	∞	∞	∞	∞	∞	∞ *5e5*	0/15
BIPOP-C	**1**(0.5)	4.9(2)	3.5(0.6)	2.2(0.3)	2.2(0.3)	2.1(0.3)	15/15
IPOP-CM	1.9(2)	**4.8**(2)	**2.7**(2)	**1.7**(1.0)*	**1.7**(1.0)*	**1.6**(0.9)*	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f8	2039	3871	4040	4219	4371	4484	15/15
NEWUOA	**1**(0.3)$^{\star 4}$	**1**(0.6)$^{\star 4}$	**1**(0.6)$^{\star 4}$	**1**(0.5)$^{\star 4}$	**1**(0.5)$^{\star 4}$	**1**(0.5)$^{\star 4}$	15/15
BIPOP-C	4.0(1)	4.0(0.7)	4.3(0.6)	4.5(0.6)	4.6(0.6)	4.6(0.6)	15/15
IPOP-CM	3.7(0.9)	3.9(0.5)	4.2(0.5)	4.4(0.4)	4.4(0.4)	4.5(0.4)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f9	1716	3102	3277	3455	3594	3727	15/15
NEWUOA	**1.0**(0.2)$^{\star 4}$	**1**(0.6)$^{\star 4}$	**1**(0.6)$^{\star 4}$	**1**(0.5)$^{\star 4}$	**1**(0.5)$^{\star 4}$	**1**(0.5)$^{\star 4}$	15/15
BIPOP-C	4.7(2)	5.7(1)	6.0(1)	6.1(1)	6.1(1.0)	6.1(0.9)	13/15
IPOP-CM	4.6(0.8)	5.7(0.5)	6.0(0.4)	6.1(0.4)	6.1(0.4)	6.1(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f10	7413	8661	10735	14920	17073	17476	15/15
NEWUOA	**1.7**(0.5)	2.6(0.8)	3.3(1)	4.0(0.8)	4.7(0.8)	5.8(1)	15/15
BIPOP-C	1.9(0.2)	1.8(0.2)	1.6(0.1)	1.2(0.0)	1.1(0.0)	1.1(0.0)	15/15
IPOP-CM	1.8(0.2)	**1.8**(0.2)	**1.5**(0.1)	**1.2**(0.1)	**1.1**(0.0)	**1.1**(0.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f11	1002	2228	6278	9762	12285	14831	15/15
NEWUOA	15(2)	13(2)	5.8(0.6)	6.1(0.5)	6.6(0.3)	6.5(0.3)	15/15
BIPOP-C	**10**(0.5)	**5.1**(0.3)	**1.9**(0.1)	**1.4**(0.0)	**1.2**(0.0)	**1.0**(0.0)	15/15
IPOP-CM	11(2)	5.4(0.9)	2.1(0.3)	1.4(0.2)	1.2(0.2)	1.1(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f12	1042	1938	2740	4140	12407	13827	15/15
NEWUOA	**3.0**(3)	**3.0**(2)	**3.0**(2)	**2.5**(1)	**1**(0.4)$^{\star 2}$	**1**(0.4)$^{\star 3}$	15/15
BIPOP-C	3.0(2)	4.0(3)	4.5(3)	4.5(2)	1.9(0.7)	2.0(0.7)	15/15
IPOP-CM	4.8(4)	5.3(5)	5.5(5)	5.1(3)	2.1(1.0)	2.2(0.9)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f13	652	2021	2751	18749	24455	30201	15/15
NEWUOA	**1**(1)$^{\star 2}$	3.0(5)	9.3(12)	19(21)	∞	∞ *2e5*	0/15
BIPOP-C	4.3(6)	**2.7**(2)	**5.1**(6)	1.5(0.8)	2.3(2)	3.0(2)	15/15
IPOP-CM	6.5(5)	4.8(5)	6.2(5)	**1.4**(0.8)	**1.7**(0.8)	**2.3**(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f14	75	239	304	932	1648	15661	15/15
NEWUOA	**1.5**(0.7)$^{\star 3}$	**1**(0.3)$^{\star 4}$	**1**(0.3)$^{\star 4}$	**1**(0.2)$^{\star 4}$	9.1(0.9)	43(32)	0/15
BIPOP-C	3.9(1)	2.9(0.4)	3.7(0.4)	4.1(0.3)	6.2(0.5)	1.2(0.1)	15/15
IPOP-CM	3.7(2)	2.8(0.5)	3.6(0.5)	3.9(0.5)	**6.0**(0.6)	**1.2**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f15	30378	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
NEWUOA	∞	∞	∞	∞	∞	∞ *1e5*	0/15
BIPOP-C	**1**(0.4)	2.0(0.8)	1.4(0.5)	1.4(0.5)	1(0.3)	1(0.3)	15/15
IPOP-CM	1.1(0.7)	**1.1**(0.4)	**0.69**(0.4)	**0.70**(0.4)	**0.52**(0.3)$_{\downarrow 2}$	**0.53**(0.3)$_{\downarrow 2}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f16	1384	27265	77015	1.9e5	2.0e5	2.2e5	15/15
NEWUOA	16(17)	∞	∞	∞	∞	∞ *2e5*	0/15
BIPOP-C	**1.7**(0.4)	1.0(0.7)	1.2(0.7)	1(0.7)	**1**(0.7)	**1**(0.7)	15/15
IPOP-CM	1.7(0.8)	**0.81**(0.6)	**0.92**(0.6)	**0.84**(0.4)	1.1(0.7)	1.0(0.6)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f17	63	1030	4005	30677	56288	80472	15/15
NEWUOA	16(4)	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP-C	2.2(2)	1(0.3)	**1**(1)	1.2(1)	1.3(0.6)	1.4(0.7)	15/15
IPOP-CM	**2.1**(1)	**0.94**(0.3)	1.2(2)	**0.76**(0.6)	**0.99**(0.3)	**1.0**(0.7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f18	621	3972	19561	67569	1.3e5	1.5e5	15/15
NEWUOA	1.2e4(1e4)	∞	∞	∞	∞	∞ *2e6*	0/15
BIPOP-C	**1.0**(0.4)	2.4(2)	1.2(0.9)	1.1(0.6)	1.7(0.7)	1.6(0.6)	15/15
IPOP-CM	1.1(0.3)	**1.8**(2)	**1.1**(0.6)	**0.97**(0.7)	**1.0**(0.4)$^{\star 2}$	**1.1**(0.4)$^{\star 2}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f19	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
NEWUOA	**76**(50)*	4.3e6(5e6)	∞	∞	∞	∞ *2e6*	0/15
BIPOP-C	169(74)	**2.4e4**(1e4)	1.2(0.6)	1(0.3)	1(0.3)	1(0.3)	15/15
IPOP-CM	161(86)	2.7e4(2e4)	**0.71**(0.5)	**0.38**(0.1)$_{\downarrow 4}$	**0.41**(0.2)$_{\downarrow 4}$	**0.41**(0.2)$_{\downarrow 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f20	82	46150	3.1e6	5.5e6	5.6e6	5.6e6	14/15
NEWUOA	**1**(0.5)$^{\star 4}$	15(17)	∞	∞	∞	∞ *4e5*	0/15
BIPOP-C	4.3(1)	9.2(4)	1(0.5)	1(0.3)	1(0.3)	1(0.3)	14/15
IPOP-CM	4.6(1)	**6.4**(2)	**0.65**(0.3)	**0.57**(0.2)	**0.58**(0.2)	**0.58**(0.2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f21	561	6541	14103	14643	15567	17589	15/15
NEWUOA	**1.7**(3)	**2.2**(2)	**1.2**(2)	**1.2**(2)	**1.1**(2)	**1**(1)	15/15
BIPOP-C	3.2(6)	55(48)	48(95)	46(93)	43(85)	39(74)	13/15
IPOP-CM	3.7(5)	139(202)	110(140)	106(136)	100(127)	88(111)	7/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f22	467	5580	23491	24948	26847	1.3e5	12/15
NEWUOA	**1**(1)*	**4.9**(6)	**6.8**(8)	**6.4**(8)	**6.0**(7)	**1.2**(1)	7/15
BIPOP-C	6.8(13)	13(21)	215(260)	202(244)	188(232)	37(46)	5/15
IPOP-CM	445(1389)	287(349)	∞	∞	∞	∞ *1e6*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f23	3.2	1614	67457	4.9e5	8.1e5	8.4e5	15/15
NEWUOA	12(8)	**3.5**(3)$^{\star 3}$	32(37)	∞	∞	∞ *2e5*	0/15
BIPOP-C	4.3(5)	32(33)	**1**(0.8)$^{\star 3}$	**2.0**(1)	**1.2**(0.9)	**1.2**(0.9)	15/15
IPOP-CM	**4.3**(6)	2.3e4(2e4)	∞	∞	∞	∞ *3e6*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
NEWUOA	∞	∞	∞	∞	∞	∞ *2e5*	0/15
BIPOP-C	**1**(0.9)	**1**(1)	**1**(1)	**1**(1)	**1**(1)	**1**(1)	3/15
IPOP-CM	∞	∞	∞	∞	∞	∞ *5e6*	0/15

Table 2: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 20. The inter-80%tile range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\text{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\text{opt}} + 10^{-8}$. Best results are printed in bold.