Bounding the Population Size of IPOP-CMA-ES on the Noiseless BBOB Testbed

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ABSTRACT

A variant of CMA-ES that uses occasional restarts coupled with an increasing population size, which is called IPOP-CMA-ES, has shown to be a top performing algorithm on the BBOB benchmark set. In this paper, we test a mechanism that bounds the maximum size that the population may reach in IPOP-CMA-ES, and we experimentally explore the impact of a maximum population size on the BBOB benchmark set. In the proposed bounding mechanism, we use a maximum population size of $10 \times D^2$, where D is problem dimension. Once the maximum population size is reached or surpassed, the population size is reset to its default starting value λ, which is defined by the $\lambda = 4 + \lfloor 3\ln(D) \rfloor$. Our experimental results show that our scheme for the population-size update can lead to improved performances on separable and weakly structured multi-modal functions.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. INTRODUCTION

IPOP-CMA-ES [1] is a variant of CMA-ES [11, 10] that uses occasional restarts, which are triggered when the search process is deemed to stagnate, combined with an increasing population size. IPOP-CMA-ES and several of its variants [12, 3, 4, 2] have shown very good results on the BBOB benchmark. In this paper, we base our analysis on IPOP-CMA-ES using its default parameter settings. In particular, the initial population size in IPOP-CMA-ES is set to $\lambda = 4 + \lfloor 3\ln(D) \rfloor$, where D is dimension of the problem being tackled. At each restart, IPOP-CMA-ES increases the population size by a factor of two. This setting leads to an exponential increase of the population size in IPOP-CMA-ES in the number of restarts. In particular, on difficult, multi-modal functions many restarts may occur and, thus, very large population sizes may result if IPOP-CMA-ES doesn't find a solution better than the optimal threshold or a possible target value.

In this paper, we use a mechanism to bound the maximum population size that IPOP-CMA-ES may use; in fact, bounding the maximum population size is motivated by the fact that sometimes very large populations may, at least theoretically, decrease the performances [5]. However, occasionally CMA-ES may benefit from large populations [9], which is also the motivation for increasing the population size in IPOP-CMA-ES. Thus, in our bound on the population size, we do not want to be too restrictive. Thus, we set the upper bound of the population size to $10 \times D^2$, which leaves for higher dimensional problems the possibility to reach rather large populations (e.g. 16 000 for $D = 40$). Once this upper bound is reached, we reset the population size to its initial value, given by $\lambda = 4 + \lfloor 3\ln(D) \rfloor$. Additionally, it gives some additional robustness with respect to the maximum bound on the population size we use. We label the resulting IPOP-CMA-ES variant IP-10DDr. The original IPOP-CMA-ES is labeled IP.

2. EXPERIMENTAL PROCEDURE

We used the C version of IPOP-CMA-ES (last modification date 10/16/10) from Hansen's webpage `http://www.lri.fr/~hansen/cmaesintro.html`. To ensure that the final best solution is inside the bounds, the bound constraints are enforced by clamping each generated solution that violates the bound constraint to the nearest solution on the bounds. The default parameter settings of IPOP-CMA-ES were used. A maximum of $10^6 \times D$ function evaluations was used for the experiments.

3. RESULTS

The results from the experiments that follow the experimental protocol [7] on the benchmark functions given in [6, 8] are presented in Figures 1, 3 and 4 and in Tables 1 and 2. The **expected running time (ERT)**, used in the figures and tables, depends on a given target function value,

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$f_t = f_{opt} + \Delta f$, and it is computed across all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [7, 13]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t (10^{-8} as in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

In the experiments, we found that IP-10DDr reaches solutions below the optimal threshold of 10^{-8} in various cases where the default version of IPOP-CMA-ES, here labeled IP, could not find such solutions. This was the case for functions f_4 ($D = 3, 5$), f_{21} ($D = 40$), f_{22} ($D = 10$) and f_{24} ($D = 2, 3$). Compared to IP, IP-10DDr uses fewer function evaluations to reach optimal threshold in functions f_3 ($D = 3, 5, 10$), f_{16} ($D = 3$), f_{21} ($D = 2, 3, 5, 10, 20$), f_{22} ($D = 2, 3, 5$), f_{23} ($D = 10, 20$); only in functions f_{19} ($D = 10$) and f_{20} ($D = 5, 10, 20$), IP-10DDr uses slightly more function evaluations to reach optimal threshold than IP.

We next examine the impact of the specific choice on the maximum population size. To do so, we explore another bound mechanism, where the maximum population size is set to a constant value of 500; a population size larger than 500 is then kept to 500. We label the resulting algorithm IP-500. Figure 1 shows that IP-10DDr clearly performs better than IP-500.

To situate the performance of IP-10DDr better with respect to other variants of IPOP-CMA-ES, in Figure 2 we show the comparisons between IP-10DDr and the performance data for the IPOP-CMA-ES variants, CMA_mah [2] and IPOPsaACM [12] in the aforementioned functions f_3, f_4,f_{16},f_{19}, f_{20}, f_{21},f_{22},f_{23},f_{24}. We find that IP-10DDr reaches the optimal threshold in functions f_4 ($D = 5$), f_{19}($D = 40$), f_{21} ($D = 40$), f_{23} ($D = 20$) and f_{24} ($D = 3$) where both, CMA_mah and IPOPsaACM, cannot reach optimal threshold. In functions f_3 ($D = 3, 5, 10$), f_4 ($D = 2, 3$), f_{16} ($D = 3$), f_{22} ($D = 3, 5$), f_{23} ($D = 3, 5, 10$) IP-10DDr uses fewer function evaluations to reach optimal threshold than CMA_mah and IPOPsaACM.

4. CPU TIMING EXPERIMENT

The IP-10DDr was run on f_8 until at least 30 seconds have passed. These experiment were conducted with Intel Xeon E5410 (2.33 GHz) on Linux (kernel 2.6.9 - 78.0.22). The results were 3.1E−05, 1.5E−05, 1.2E−05, 9.5E−06, 1.5E−05 and 5.0E−05 seconds per function evaluation in dimensions 2, 3, 5, 10, 20, and 40, respectively.

5. CONCLUSIONS

In this paper, we have studied the impact of bounding the population size in IPOP-CMA-ES together with re-initialization of the population size. Obviously, using a maximum population size of $10 \times D^2$ does not worsen results on functions that are easy for IPOP-CMA-ES, that is, on functions where IPOP-CMA-ES within the first trial or very few restarts finds the optimum–in such cases the bounds do not take effect. However, for various difficult, multi-modal functions we observed improved performance of our new IPOP-CMA-ES variants over the default IPOP-CMA-ES. Hence, these results would encourage us to explore bounds on the maximum population size also for other IPOP-CMA-ES variants such as Bipop-CMA-ES. Finally, one may further explore different settings for the bounds on the maximum population size, which may lead to further improvements in performance.

6. ACKNOWLEDGMENTS

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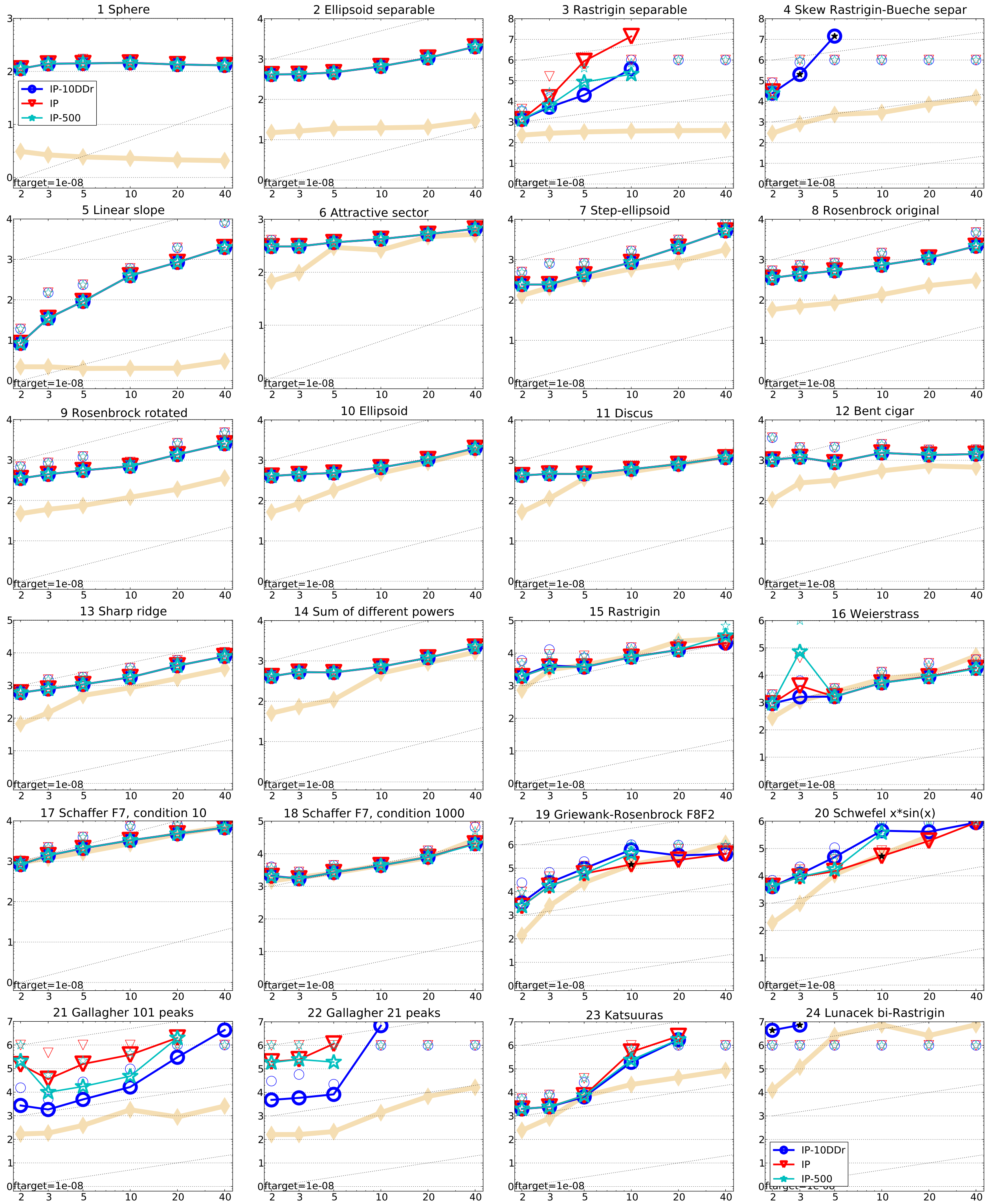

Figure 1: Expected running time (ERT in number of f-evaluations) divided by dimension for target function value 10^{-8} as $\log_{10}$ values versus dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: $\circ$:IP-10DDr, $\triangledown$:IP, $\star$:IP-500

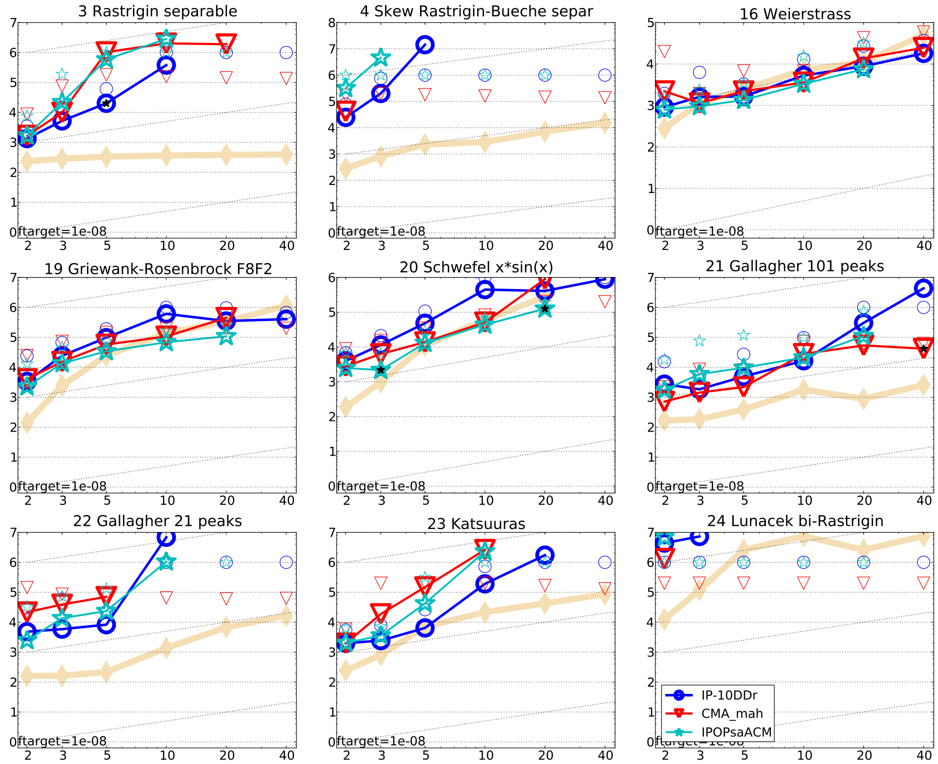

Figure 2: Comparing IP-10DDr to CMA_mah and IPOPsaACM. CMA_mah is the active covariance matrix adaptation version of IPOP-CMA-ES with mirrored mutation and a small initial population size. CMA_sa is self-adaptive surrogate-assisted version of IPOP-CMA-ES. Expected running time (ERT) divided by dimension for target function value 10^{-8} as $\log_{10}$ values. Different symbols correspond to different algorithms given in legend of f_{24}. Light symbols give the maximum number of function evaluations from all trials divided by the dimension. Horizontal lines give linear scaling, the slanted dotted lines give quadratic scaling.

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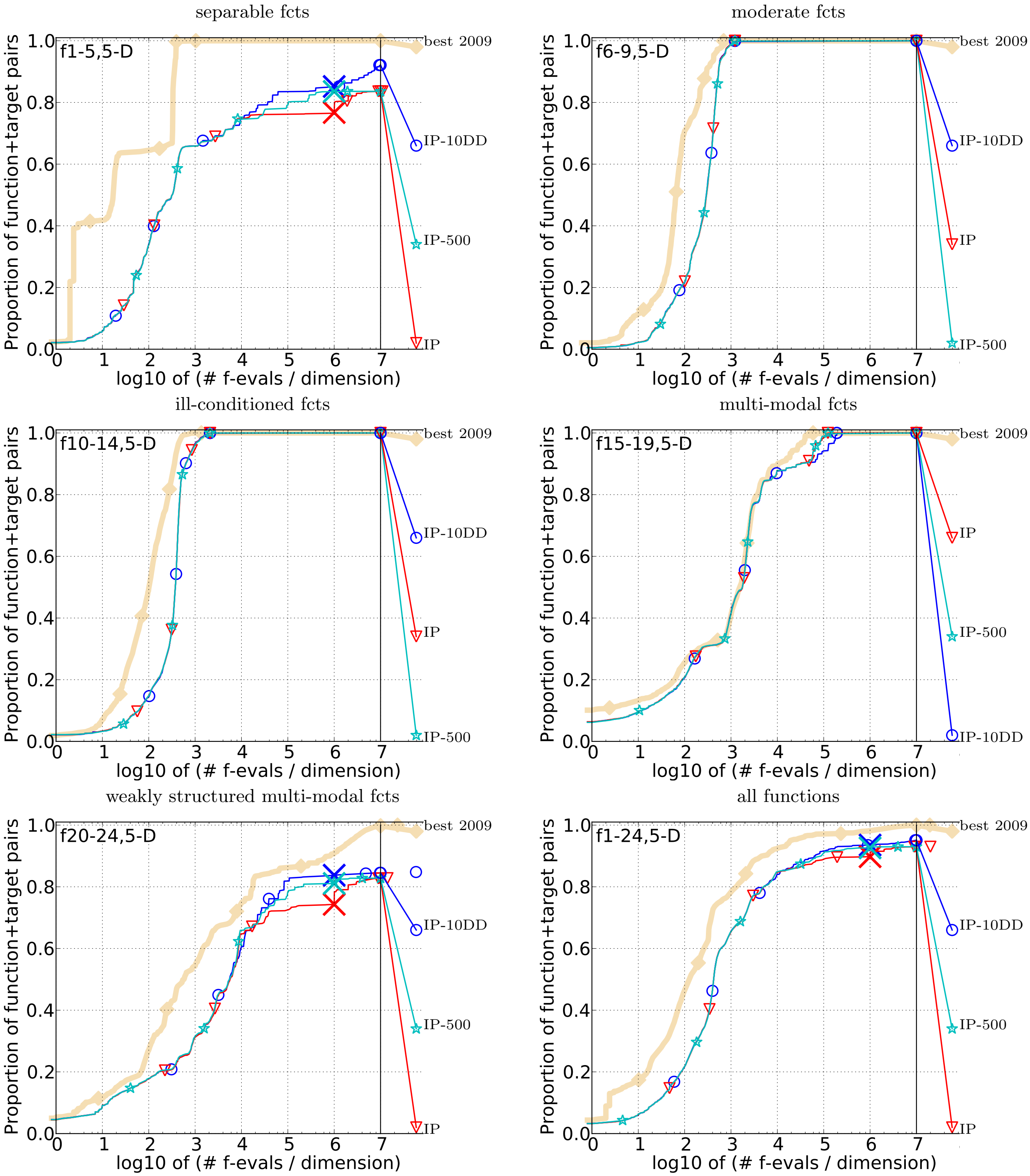

Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

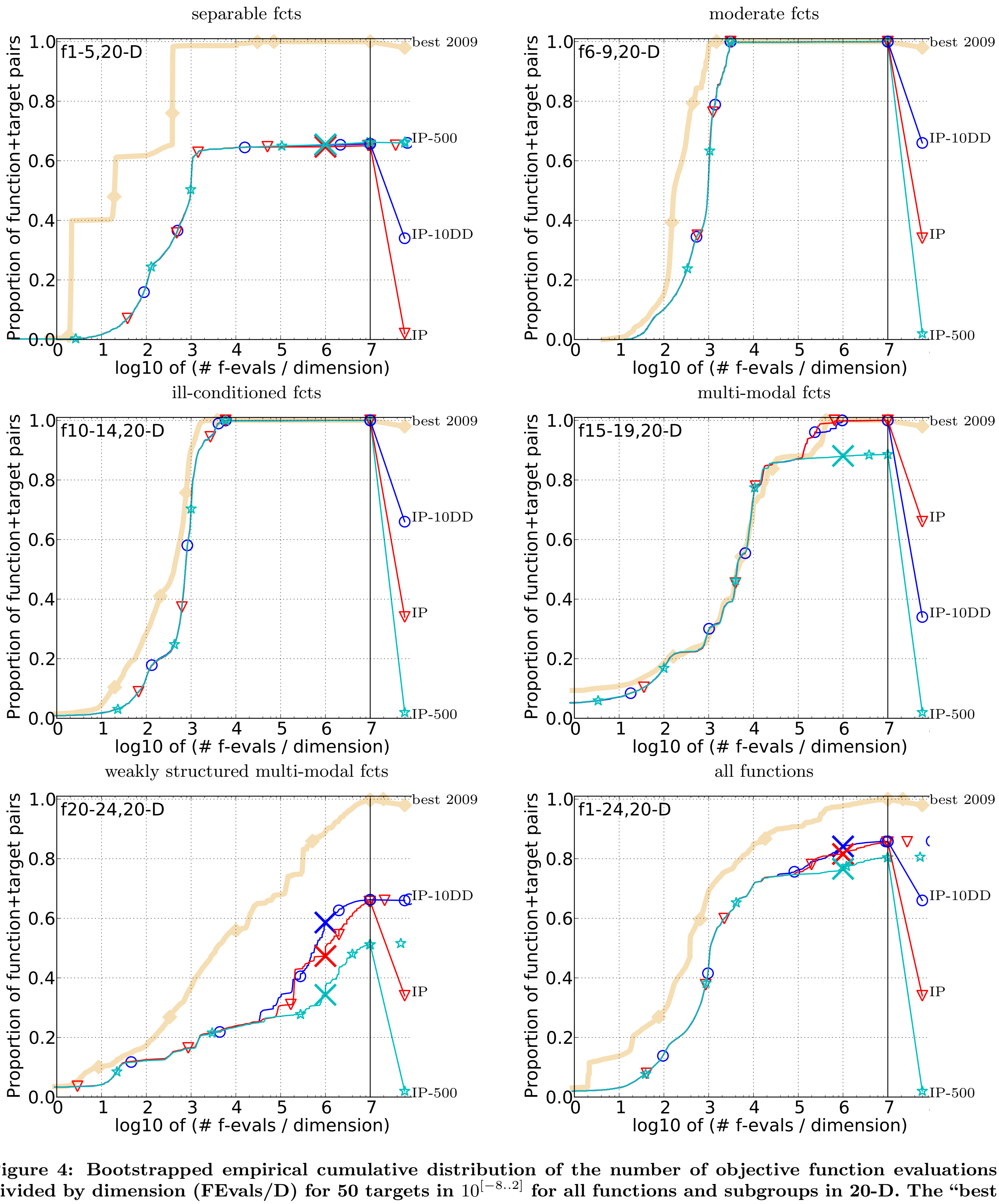

Figure 4: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	15/15
IP-10DD	**3.0**(1)	**7.5**(1)	**13**(3)	**27**(3)	**40**(4)	**53**(6)	15/15
IP	**3.0**(1)	**7.5**(1)	**13**(3)	**27**(3)	**40**(4)	**53**(6)	15/15
IP-500	**3.0**(1)	**7.5**(1)	**13**(3)	**27**(3)	**40**(4)	**53**(6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f2	83	87	88	90	92	94	15/15
IP-10DD	**13**(4)	**17**(5)	**19**(4)	**21**(3)	**23**(3)	**24**(2)	15/15
IP	**13**(4)	**17**(5)	**19**(4)	**21**(3)	**23**(3)	**24**(2)	15/15
IP-500	**13**(4)	**17**(5)	**19**(4)	**21**(3)	**23**(3)	**24**(2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f3	716	1622	1637	1646	1650	1654	15/15
IP-10DD	**0.99**(1)	**15**(21)	**60**(67)	**60**(67)	**61**(67)	**61**(67)	15/15
IP	**0.99**(1)	48(11)	2690(3070)	2677(4559)	2670(4543)	2665(3043)	8/15
IP-500	**0.99**(1)	18(11)	257(433)	258(433)	259(432)	259(432)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4	809	1633	1688	1817	1886	1903	15/15
IP-10DD	**2.6**(3)	**2721**(2469)$^{\star 3}$	**4.3e4**(5e4)$^{\star 3}$	**4.0e4**(5e4)$^{\star 3}$	**3.8e4**(4e4)$^{\star 3}$	**3.8e4**(5e4)$^{\star 3}$	1/15
IP	**2.6**(3)	∞	∞	∞	∞	∞ *5e6*	0/15
IP-500	**2.6**(3)	∞	∞	∞	∞	∞ *5e6*	0/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f5	10	10	10	10	10	10	15/15
IP-10DD	**6.2**(3)	**23**(18)	**30**(25)	**41**(33)	**45**(43)	**46**(43)	15/15
IP	**6.2**(3)	**23**(18)	**30**(25)	**41**(33)	**45**(43)	**46**(43)	15/15
IP-500	**6.2**(3)	**23**(18)	**30**(25)	**41**(33)	**45**(43)	**46**(43)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f6	114	214	281	580	1038	1332	15/15
IP-10DD	**1.6**(0.5)	**1.8**(0.4)	**2.1**(0.3)	**1.6**(0.2)	**1.2**(0.1)	**1.2**(0.1)	15/15
IP	**1.6**(0.5)	**1.8**(0.4)	**2.1**(0.3)	**1.6**(0.2)	**1.2**(0.1)	**1.2**(0.1)	15/15
IP-500	**1.6**(0.5)	**1.8**(0.4)	**2.1**(0.3)	**1.6**(0.2)	**1.2**(0.1)	**1.2**(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f7	24	324	1171	1572	1572	1597	15/15
IP-10DD	**5.3**(2)	**1.1**(0.4)	**1.2**(0.9)	**1.2**(0.7)	**1.2**(0.7)	**1.3**(0.8)	15/15
IP	**5.3**(2)	**1.1**(0.4)	**1.2**(0.9)	**1.2**(0.7)	**1.2**(0.7)	**1.3**(0.8)	15/15
IP-500	**5.3**(2)	**1.1**(0.4)	**1.2**(0.9)	**1.2**(0.7)	**1.2**(0.7)	**1.3**(0.8)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f8	73	273	336	391	410	422	15/15
IP-10DD	**3.2**(1.0)	**4.3**(2)	**5.1**(1)	**5.5**(1)	**5.8**(1)	**6.1**(1)	15/15
IP	**3.2**(1.0)	**4.3**(2)	**5.1**(1)	**5.5**(1)	**5.8**(1)	**6.1**(1)	15/15
IP-500	**3.2**(1.0)	**4.3**(2)	**5.1**(1)	**5.5**(1)	**5.8**(1)	**6.1**(1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f9	35	127	214	300	335	369	15/15
IP-10DD	**6.1**(2)	**10**(10)	**8.6**(6)	**7.6**(5)	**7.4**(4)	**7.2**(4)	15/15
IP	**6.1**(2)	**10**(10)	**8.6**(6)	**7.6**(5)	**7.4**(4)	**7.2**(4)	15/15
IP-500	**6.1**(2)	**10**(10)	**8.6**(6)	**7.6**(5)	**7.4**(4)	**7.2**(4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f10	349	500	574	626	829	880	15/15
IP-10DD	**3.4**(0.9)	**3.1**(0.6)	**3.1**(0.3)	**3.2**(0.3)	**2.6**(0.2)	**2.7**(0.2)	15/15
IP	**3.4**(0.9)	**3.1**(0.6)	**3.1**(0.3)	**3.2**(0.3)	**2.6**(0.2)	**2.7**(0.2)	15/15
IP-500	**3.4**(0.9)	**3.1**(0.6)	**3.1**(0.3)	**3.2**(0.3)	**2.6**(0.2)	**2.7**(0.2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f11	143	202	763	1177	1467	1673	15/15
IP-10DD	**7.4**(4)	**7.1**(0.7)	**2.1**(0.2)	**1.6**(0.1)	**1.4**(0.1)	**1.3**(0.1)	15/15
IP	**7.4**(4)	**7.1**(0.7)	**2.1**(0.2)	**1.6**(0.1)	**1.4**(0.1)	**1.3**(0.1)	15/15
IP-500	**7.4**(4)	**7.1**(0.7)	**2.1**(0.2)	**1.6**(0.1)	**1.4**(0.1)	**1.3**(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f12	108	268	371	461	1303	1494	15/15
IP-10DD	**7.9**(5)	**5.9**(3)	**6.0**(3)	**6.3**(4)	**2.8**(2)	**2.8**(2)	15/15
IP	**7.9**(5)	**5.9**(3)	**6.0**(3)	**6.3**(4)	**2.8**(2)	**2.8**(2)	15/15
IP-500	**7.9**(5)	**5.9**(3)	**6.0**(3)	**6.3**(4)	**2.8**(2)	**2.8**(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f13	132	195	250	1310	1752	2255	15/15
IP-10DD	**4.7**(4)	**5.8**(4)	**5.7**(3)	**1.5**(0.7)	**1.9**(0.8)	**1.9**(0.8)	15/15
IP	**4.7**(4)	**5.8**(4)	**5.7**(3)	**1.5**(0.7)	**1.9**(0.8)	**1.9**(0.8)	15/15
IP-500	**4.7**(4)	**5.8**(4)	**5.7**(3)	**1.5**(0.7)	**1.9**(0.8)	**1.9**(0.8)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f14	10	41	58	139	251	476	15/15
IP-10DD	**1.2**(1)	**2.4**(1)	**3.7**(1.0)	**4.7**(0.9)	**5.7**(0.4)	**4.6**(0.4)	15/15
IP	**1.2**(1)	**2.4**(1)	**3.7**(1.0)	**4.7**(0.9)	**5.7**(0.4)	**4.6**(0.4)	15/15
IP-500	**1.2**(1)	**2.4**(1)	**3.7**(1.0)	**4.7**(0.9)	**5.7**(0.4)	**4.6**(0.4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f15	511	9310	19369	20073	20769	21359	14/15
IP-10DD	**1.5**(0.6)	**1.1**(0.9)	**0.85**(0.4)	**0.85**(0.4)	**0.85**(0.4)	**0.86**(0.4)	15/15
IP	**1.5**(0.6)	**1.1**(0.9)	0.86(0.4)	0.86(0.4)	0.87(0.4)	0.87(0.4)	15/15
IP-500	**1.5**(0.6)	**1.1**(0.9)	0.86(0.4)	0.86(0.4)	0.87(0.4)	0.87(0.4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f16	120	612	2662	10449	11644	12095	15/15
IP-10DD	**2.0**(2)	**2.2**(3)	**1.1**(1)	**0.58**(0.5)	**0.67**(0.3)	**0.67**(0.3)	15/15
IP	**2.0**(2)	**2.2**(3)	**1.1**(1)	**0.58**(0.5)	**0.67**(0.3)	**0.67**(0.3)	15/15
IP-500	**2.0**(2)	**2.2**(3)	**1.1**(1)	**0.58**(0.5)	**0.67**(0.3)	**0.67**(0.3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f17	5.2	215	899	3669	6351	7934	15/15
IP-10DD	**3.2**(3)	**0.79**(0.2)	**0.77**(0.2)	**0.75**(0.5)	**1.1**(0.4)	**1.3**(0.4)	15/15
IP	**3.2**(3)	**0.79**(0.2)	**0.77**(0.2)	**0.75**(0.5)	**1.1**(0.4)	**1.3**(0.4)	15/15
IP-500	**3.2**(3)	**0.79**(0.2)	**0.77**(0.2)	**0.75**(0.5)	**1.1**(0.4)	**1.3**(0.4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f18	103	378	3968	9280	10905	12469	15/15
IP-10DD	**1.1**(0.4)	**1.8**(0.3)	**0.56**(0.5)	**1.0**(0.3)	**0.97**(0.3)	**1.0**(0.1)	15/15
IP	**1.1**(0.4)	**1.8**(0.3)	**0.56**(0.5)	**1.0**(0.3)	**0.97**(0.3)	**1.0**(0.1)	15/15
IP-500	**1.1**(0.4)	**1.8**(0.3)	**0.56**(0.5)	**1.0**(0.3)	**0.97**(0.3)	**1.0**(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f19	1	1	242	1.2e5	1.2e5	1.2e5	15/15
IP-10DD	**13**(14)	**1466**(1310)	519(658)	4.0(3)	4.0(3)	4.0(3)	15/15
IP	**13**(14)	**1466**(1310)	**483**(576)	2.4(2)	2.4(2)	2.4(2)	15/15
IP-500	**13**(14)	**1466**(1310)	**483**(576)	**2.4**(2)	**2.4**(2)	**2.4**(2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f20	16	851	38111	54470	54861	55313	14/15
IP-10DD	**3.4**(1)	**12**(11)	6.2(5)	4.4(4)	4.4(4)	4.4(4)	15/15
IP	**3.4**(1)	**12**(11)	**1.7**(2)	**1.3**(1)	**1.3**(1)	**1.3**(1)	15/15
IP-500	**3.4**(1)	**12**(11)	2.1(3)	1.5(2)	1.5(2)	1.5(2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f21	41	1157	1674	1705	1729	1757	14/15
IP-10DD	**1.6**(1)	11(12)	**14**(18)	**14**(18)	**14**(17)	**14**(17)	15/15
IP	**1.6**(1)	8.7(9)	290(491)	458(1467)	452(1446)	445(1424)	13/15
IP-500	**1.6**(1)	**8.6**(9)	51(11)	50(12)	50(12)	50(12)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f22	71	386	938	1008	1040	1068	14/15
IP-10DD	**7.4**(11)	42(53)	**43**(59)	**40**(55)	**39**(53)	**39**(52)	15/15
IP	**7.4**(11)	**33**(27)	1315(2668)	5753(7455)	5578(7225)	5438(7028)	7/15
IP-500	**7.4**(11)	92(27)	580(280)	962(2534)	933(2404)	910(2344)	13/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f23	3.0	518	14249	31654	33030	34256	15/15
IP-10DD	**2.1**(2)	**7.2**(5)	**1.7**(1)	**0.96**(0.8)	**0.94**(0.8)	**0.93**(0.8)	15/15
IP	**2.1**(2)	**7.2**(5)	2.0(1)	1.1(0.8)	1.1(0.8)	1.1(0.8)	15/15
IP-500	**2.1**(2)	**7.2**(5)	2.0(1)	1.1(0.8)	1.1(0.8)	1.1(0.8)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f24	1622	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
IP-10DD	**2.0**(2)	**104**(111)$^{\star 4}$	∞	∞	∞	∞ *5e6*	0/15
IP	**2.0**(2)	∞	∞	∞	∞	∞ *5e6*	0/15
IP-500	**2.0**(2)	∞	∞	∞	∞	∞ *5e6*	0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 5. The central 80% range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Best results are printed in bold. . IP-10DD in the table denotes IP-10DDr.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	15/15
IP-10DD	**7.5**(0.6)	**14**(0.7)	**20**(2)	**32**(3)	**45**(2)	**57**(3)	15/15
IP	**7.5**(0.6)	**14**(0.7)	**20**(2)	**32**(3)	**45**(2)	**57**(3)	15/15
IP-500	**7.5**(0.6)	**14**(0.7)	**20**(2)	**32**(3)	**45**(2)	**57**(3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f2	385	386	387	390	391	393	15/15
IP-10DD	**35**(4)	**42**(4)	**48**(4)	**51**(2)	**52**(2)	**53**(2)	15/15
IP	**35**(4)	**42**(4)	**48**(4)	**51**(2)	**52**(2)	**53**(2)	15/15
IP-500	**35**(4)	**42**(4)	**48**(4)	**51**(2)	**52**(2)	**53**(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f3	5066	7626	7635	7643	7646	7651	15/15
IP-10DD	**15**(16)	∞	∞	∞	∞	∞ *2e7*	0/15
IP	**15**(16)	∞	∞	∞	∞	∞ *2e7*	0/15
IP-500	**15**(16)	**1.9e4**(2e4)	∞	∞	∞	∞ *2e7*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4	4722	7628	7666	7700	7758	1.4e5	9/15
IP-10DD	**2.9e4**(3e4)	∞	∞	∞	∞	∞ *2e7*	0/15
IP	∞	∞	∞	∞	∞	∞ *2e7*	0/15
IP-500	∞	∞	∞	∞	∞	∞ *2e7*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f5	41	41	41	41	41	41	15/15
IP-10DD	**51**(35)	**111**(93)	**165**(106)	**233**(151)	**309**(245)	**403**(293)	15/15
IP	**51**(35)	**111**(93)	**165**(106)	**233**(151)	**309**(245)	**403**(293)	15/15
IP-500	**51**(35)	**111**(93)	**165**(106)	**233**(151)	**309**(245)	**403**(293)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f6	1296	2343	3413	5220	6728	8409	15/15
IP-10DD	**1.3**(0.2)	**1.1**(0.2)	**1.1**(0.2)	**1.1**(0.1)	**1.1**(0.1)	**1.1**(0.1)	15/15
IP	**1.3**(0.2)	**1.1**(0.2)	**1.1**(0.2)	**1.1**(0.1)	**1.1**(0.1)	**1.1**(0.1)	15/15
IP-500	**1.3**(0.2)	**1.1**(0.2)	**1.1**(0.2)	**1.1**(0.1)	**1.1**(0.1)	**1.1**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f7	1351	4274	9503	16524	16524	16969	15/15
IP-10DD	**1.7**(2)	**6.2**(3)	**3.9**(2)	**2.4**(1.0)	**2.4**(1.0)	**2.4**(1.0)	15/15
IP	**1.7**(2)	**6.2**(3)	**3.9**(2)	**2.4**(1.0)	**2.4**(1.0)	**2.4**(1.0)	15/15
IP-500	**1.7**(2)	**6.2**(3)	**3.9**(2)	**2.4**(1.0)	**2.4**(1.0)	**2.4**(1.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f8	2039	3871	4040	4219	4371	4484	15/15
IP-10DD	**4.1**(0.8)	**4.3**(0.3)	**4.6**(0.4)	**4.8**(0.3)	**4.8**(0.3)	**4.9**(0.3)	15/15
IP	**4.1**(0.8)	**4.3**(0.3)	**4.6**(0.4)	**4.8**(0.3)	**4.8**(0.3)	**4.9**(0.3)	15/15
IP-500	**4.1**(0.8)	**4.3**(0.3)	**4.6**(0.4)	**4.8**(0.3)	**4.8**(0.3)	**4.9**(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f9	1716	3102	3277	3455	3594	3727	15/15
IP-10DD	**5.0**(1)	**6.9**(5)	**7.2**(5)	**7.3**(4)	**7.3**(4)	**7.2**(4)	15/15
IP	**5.0**(1)	**6.9**(5)	**7.2**(5)	**7.3**(4)	**7.3**(4)	**7.2**(4)	15/15
IP-500	**5.0**(1)	**6.9**(5)	**7.2**(5)	**7.3**(4)	**7.3**(4)	**7.2**(4)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f10	7413	8661	10735	14920	17073	17476	15/15
IP-10DD	**1.8**(0.2)	**1.9**(0.2)	**1.7**(0.1)	**1.3**(0.0)	**1.2**(0.0)	**1.2**(0.0)	15/15
IP	**1.8**(0.2)	**1.9**(0.2)	**1.7**(0.1)	**1.3**(0.0)	**1.2**(0.0)	**1.2**(0.0)	15/15
IP-500	**1.8**(0.2)	**1.9**(0.2)	**1.7**(0.1)	**1.3**(0.0)	**1.2**(0.0)	**1.2**(0.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f11	1002	2228	6278	9762	12285	14831	15/15
IP-10DD	**10**(0.6)	**5.1**(0.2)	**2.0**(0.1)	**1.4**(0.0)	**1.2**(0.0)	**1.1**(0.0)	15/15
IP	**10**(0.6)	**5.1**(0.2)	**2.0**(0.1)	**1.4**(0.0)	**1.2**(0.0)	**1.1**(0.0)	15/15
IP-500	**10**(0.6)	**5.1**(0.2)	**2.0**(0.1)	**1.4**(0.0)	**1.2**(0.0)	**1.1**(0.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f12	1042	1938	2740	4140	12407	13827	15/15
IP-10DD	**2.6**(2)	**3.6**(3)	**4.1**(3)	**4.1**(2)	**1.8**(0.6)	**1.9**(0.5)	15/15
IP	**2.6**(2)	**3.6**(3)	**4.1**(3)	**4.1**(2)	**1.8**(0.6)	**1.9**(0.5)	15/15
IP-500	**2.6**(2)	**3.6**(3)	**4.1**(3)	**4.1**(2)	**1.8**(0.6)	**1.9**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f13	652	2021	2751	18749	24455	30201	15/15
IP-10DD	**2.4**(0.3)	**4.3**(5)	**6.4**(4)	**1.7**(1)	**2.2**(0.9)	**2.3**(0.9)	15/15
IP	**2.4**(0.3)	**4.3**(5)	**6.4**(4)	**1.7**(1)	**2.2**(0.9)	**2.3**(0.9)	15/15
IP-500	**2.4**(0.3)	**4.3**(5)	**6.4**(4)	**1.7**(1)	**2.2**(0.9)	**2.3**(0.9)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f14	75	239	304	932	1648	15661	15/15
IP-10DD	**3.8**(0.7)	**2.7**(0.5)	**3.4**(0.4)	**4.2**(0.4)	**6.3**(0.4)	**1.2**(0.1)	15/15
IP	**3.8**(0.7)	**2.7**(0.5)	**3.4**(0.4)	**4.2**(0.4)	**6.3**(0.4)	**1.2**(0.1)	15/15
IP-500	**3.8**(0.7)	**2.7**(0.5)	**3.4**(0.4)	**4.2**(0.4)	**6.3**(0.4)	**1.2**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f15	30378	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
IP-10DD	**1.1**(0.7)	**0.99**(0.5)	**0.70**(0.2)	**0.71**(0.2)	**0.53**(0.2)$\downarrow_2$	**0.54**(0.2)$\downarrow_2$	15/15
IP	**1.1**(0.7)	**0.99**(0.5)	**0.70**(0.2)	**0.71**(0.2)	**0.53**(0.2)$\downarrow_2$	**0.54**(0.2)$\downarrow_2$	15/15
IP-500	**1.1**(0.7)	0.99(0.5)	0.71(0.2)	0.72(0.2)	0.53(0.1)$\downarrow_3$	0.54(0.1)$\downarrow_3$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f16	1384	27265	77015	1.9e5	2.0e5	2.2e5	15/15
IP-10DD	**1.2**(0.3)	**0.27**(0.3)	**0.42**(0.3)$\downarrow_2$	**0.78**(0.6)	0.85(0.6)	0.79(0.5)	15/15
IP	**1.2**(0.3)	**0.27**(0.3)	**0.42**(0.3)$\downarrow_2$	**0.78**(0.6)	0.85(0.6)	0.79(0.5)	15/15
IP-500	**1.2**(0.3)	**0.27**(0.3)	**0.42**(0.3)$\downarrow_2$	**0.78**(0.6)	**0.83**(0.6)	**0.77**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f17	63	1030	4005	30677	56288	80472	15/15
IP-10DD	**1.9**(0.8)	**0.78**(0.1)$\downarrow$	**0.67**(0.1)	**0.93**(0.4)	**1.1**(0.4)	**1.1**(0.5)	15/15
IP	**1.9**(0.8)	**0.78**(0.1)$\downarrow$	**0.67**(0.1)	**0.93**(0.4)	**1.1**(0.4)	**1.1**(0.5)	15/15
IP-500	**1.9**(0.8)	**0.78**(0.1)$\downarrow$	**0.67**(0.1)	**0.93**(0.4)	**1.1**(0.4)	**1.1**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f18	621	3972	19561	67569	1.3e5	1.5e5	15/15
IP-10DD	**0.87**(0.2)	**0.46**(0.2)	**0.66**(0.4)	**1.2**(0.8)	**0.96**(0.4)	**1.0**(0.4)	15/15
IP	**0.87**(0.2)	**0.46**(0.2)	**0.66**(0.4)	**1.2**(0.8)	**0.96**(0.4)	**1.0**(0.4)	15/15
IP-500	**0.87**(0.2)	**0.46**(0.2)	**0.66**(0.4)	**1.2**(0.8)	**0.96**(0.4)	**1.0**(0.4)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f19	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
IP-10DD	**178**(144)	**5.2e5**(2e5)	**4.0**(3)	0.84(0.6)	1.0(0.9)	1.0(0.9)	15/15
IP	**178**(144)	**5.2e5**(2e5)	**4.0**(3)	**0.67**(0.3)	**0.66**(0.3)	**0.66**(0.3)	15/15
IP-500	**178**(144)	**5.2e5**(2e5)	**4.0**(3)	∞	∞	∞ *2e7*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f20	82	46150	3.1e6	5.5e6	5.6e6	5.6e6	14/15
IP-10DD	**5.4**(1)	**5.6**(3)	0.91(0.6)	1.5(1)	1.5(1)	1.5(1)	15/15
IP	**5.4**(1)	**5.6**(3)	**0.86**(0.4)	**0.66**(0.2)	**0.66**(0.2)	**0.67**(0.2)	15/15
IP-500	**5.4**(1)	5.8(3)	∞	∞	∞	∞ *2e7*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f21	561	6541	14103	14643	15567	17589	15/15
IP-10DD	**6.3**(11)	**147**(189)	**430**(665)	**414**(641)	**390**(603)	**345**(533)	14/15
IP	**6.3**(11)	924(1530)	2859(3545)	2755(3415)	2592(3264)	2295(2890)	5/15
IP-500	**6.3**(11)	856(1529)	2850(3545)	2745(3444)	2582(3240)	2285(3411)	5/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f22	467	5580	23491	24948	26847	1.3e5	12/15
IP-10DD	**96**(168)	**1871**(3584)	∞	∞	∞	∞ *2e7*	0/15
IP	**96**(168)	2411(3592)	∞	∞	∞	∞ *2e7*	0/15
IP-500	525(247)	2405(3589)	∞	∞	∞	∞ *2e7*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f23	3.2	1614	67457	4.9e5	8.1e5	8.4e5	15/15
IP-10DD	**1.7**(2)	**35**(39)	**5.3**(5)	**71**(66)	**43**(45)	**41**(43)	7/15
IP	**1.7**(2)	**35**(39)	**5.3**(5)	100(102)	60(63)	58(62)	5/15
IP-500	**1.7**(2)	**35**(39)	**5.3**(5)	72(70)	44(42)	42(41)	7/15

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
IP-10DD	∞	∞	∞	∞	∞	∞ *2e7*	0/15
IP	∞	∞	∞	∞	∞	∞ *2e7*	0/15
IP-500	∞	∞	∞	∞	∞	∞ *2e7*	0/15

Table 2: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 20. The central 80% range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Best results are printed in bold. . IP-10DD in the table denotes IP-10DDr.