

# Hypervolume-based DIRECT

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# DIRECT (Diving RECTangles) Framework

Splits the solution space in the form of a hyperrectangle into smaller hyperrectangles

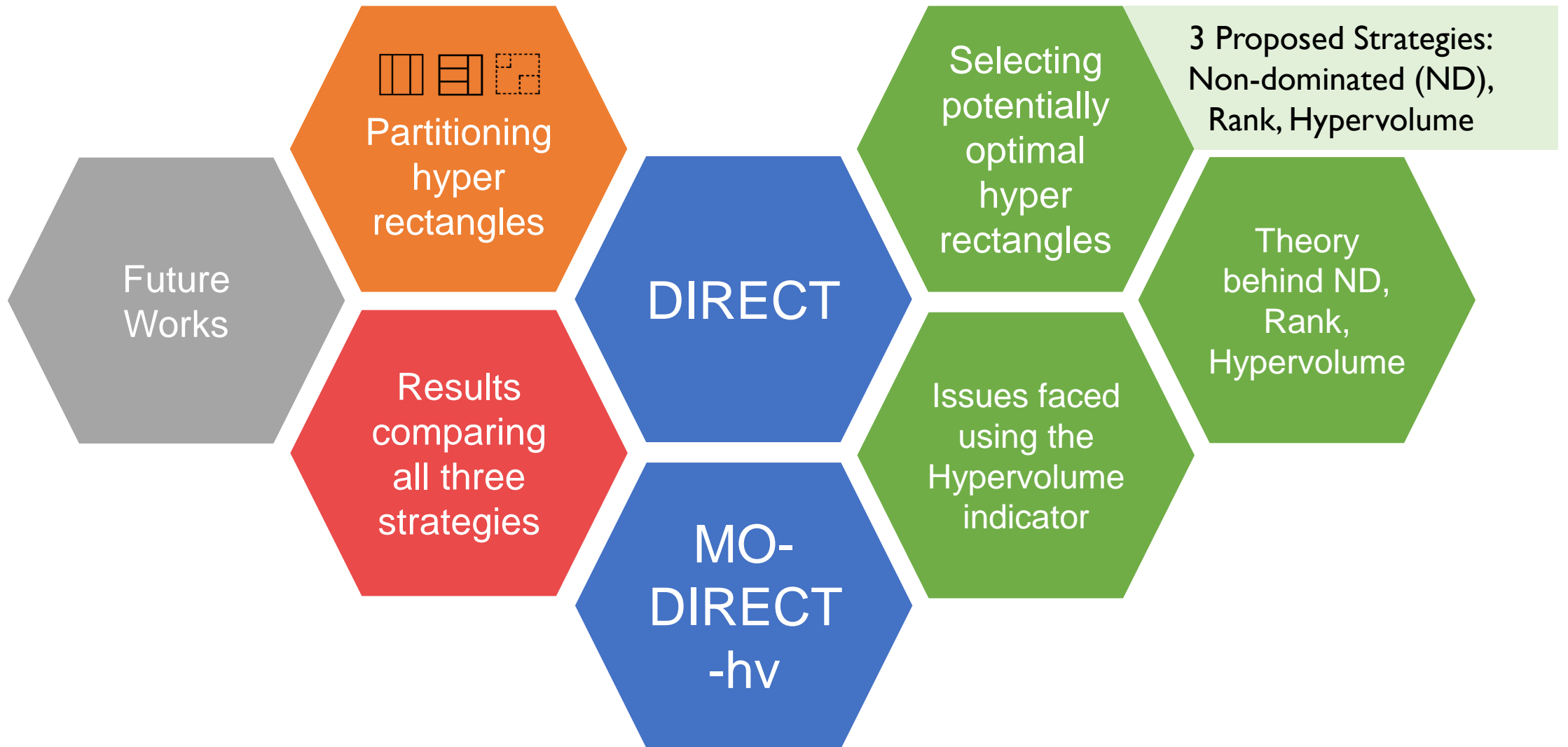


Partitioning  
hyper  
rectangles

DIRECT

Selecting  
potentially  
optimal  
hyper  
rectangles

Using solution quality indicators and the size of the hyper rectangles to identify potentially optimal areas in solution space

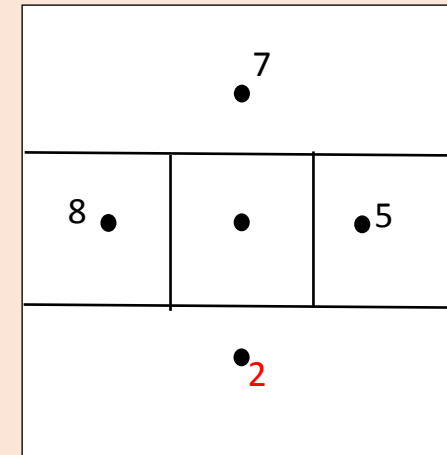
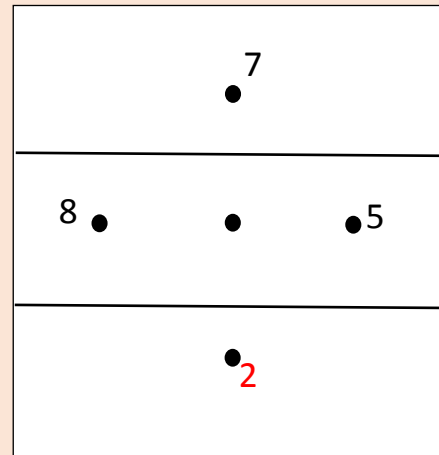
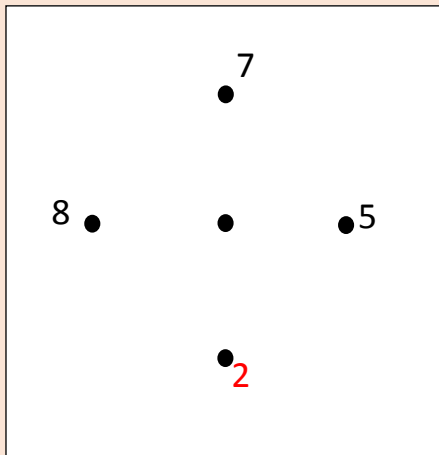




Partitioning  
hyper  
rectangles

Basic Rule: Start partitioning with the best objective function value  
Rationale: They will be bigger size rectangles, hence more likely to  
be chosen as potentially optimal hyperrectangles

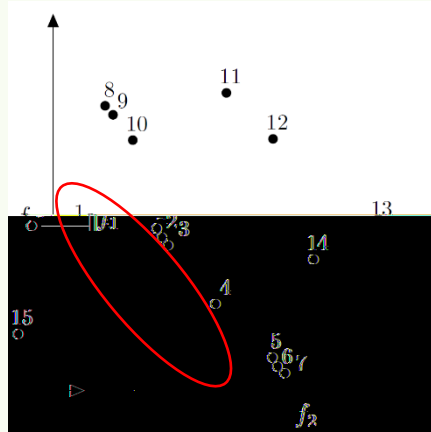
2D example



## Non-dominated (ND)

Selecting hyperrectangles based on

- Non-dominated solutions
- Size of hyperrectangles

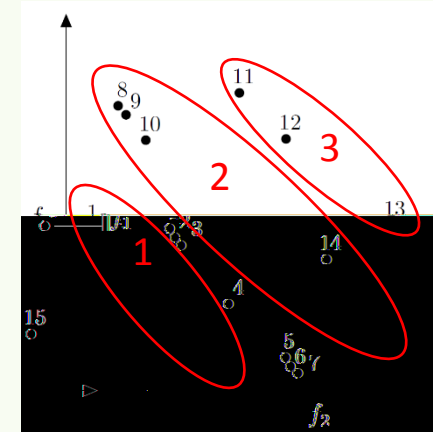


# Selecting potentially optimal hyperrectangles

## Rank

Selecting hyperrectangles based on

- Rank (Different fronts)
- Size of hyperrectangles



## Hypervolume (HV)

Selecting hyperrectangles based on

- Hypervolume
- Size of hyperrectangles

Hypervolume:

Quality indicator of non-dominated solutions  
Measures the diversity of the solution through the area of space it occupies

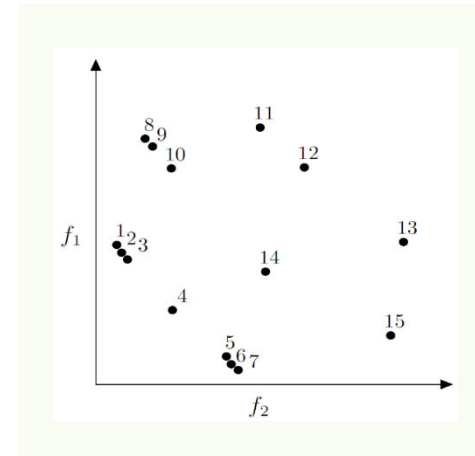
# Non-dominated solutions

ND: 1,2,3,4,5,6,7

Rank: 7 (largest hyperrectangle)

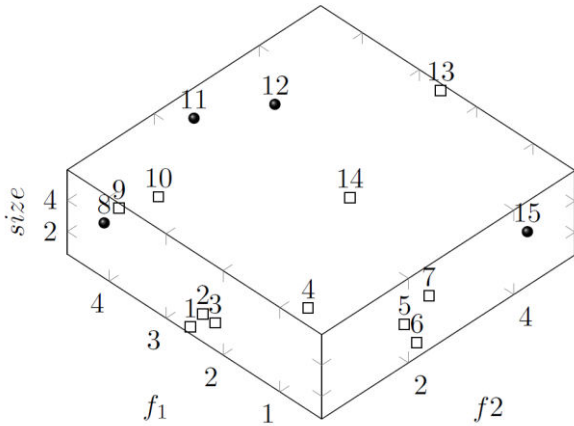
HV: 4 and 7 (4 has large hypervolume because lack of other non-dominated solutions around it)

Using hypervolume to select potentially optimal hyperrectangles allows careful selection on the non-dominated front

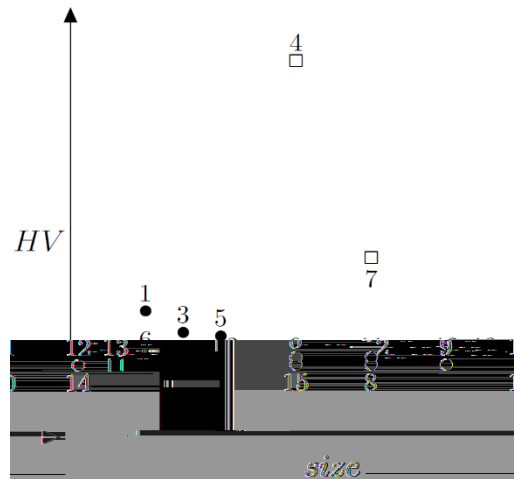


- Chosen as potentially optimal point
- Not chosen as potentially optimal point

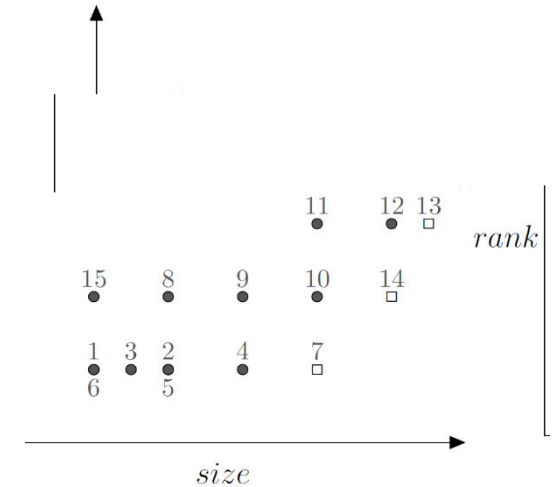
## ND strategy

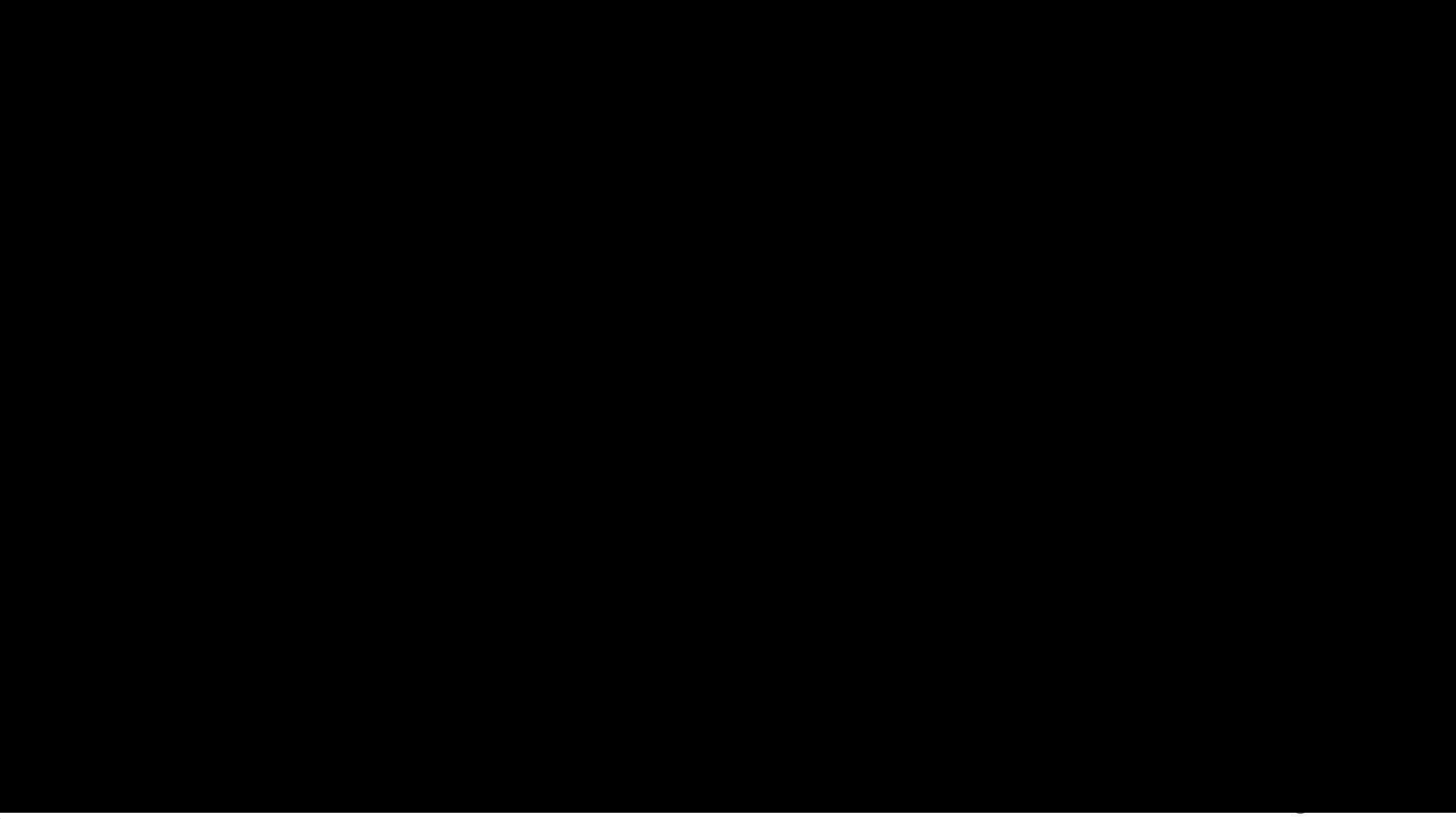


## Hypervolume

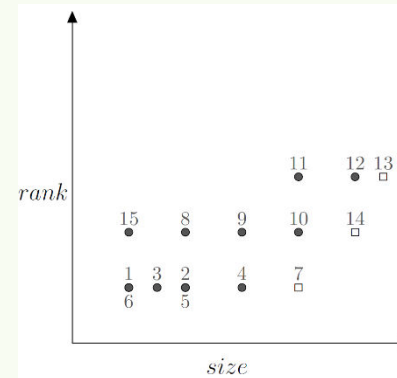
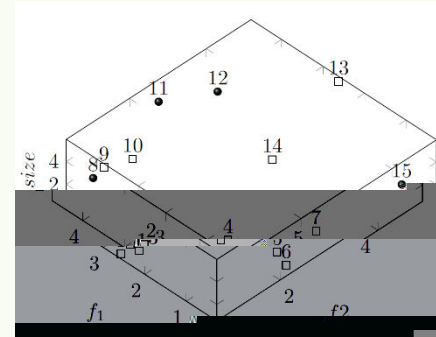


## Rank





Consider other strategies...



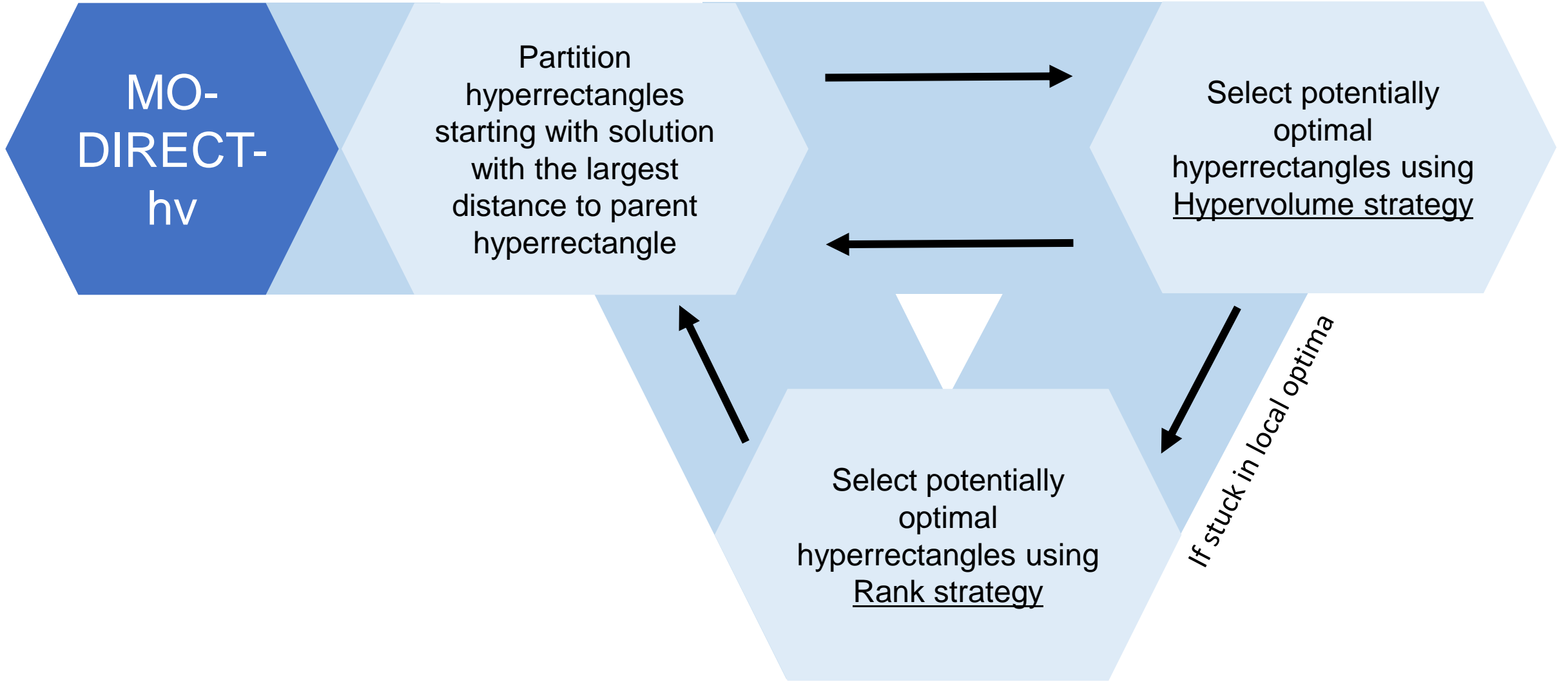
Rank  
Strategy  
used to  
explore  
unexplored  
space

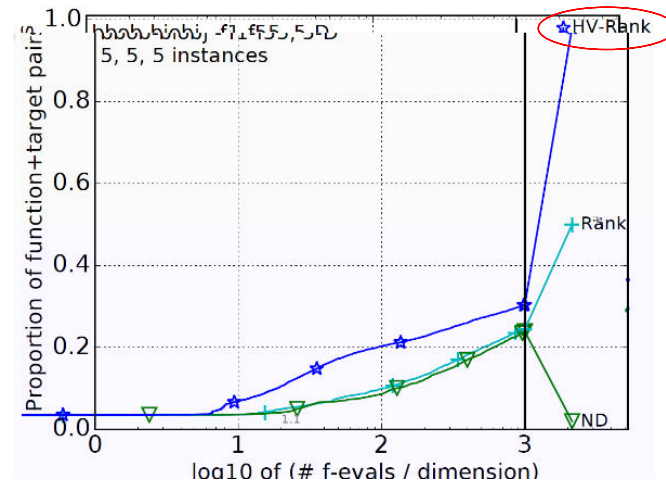
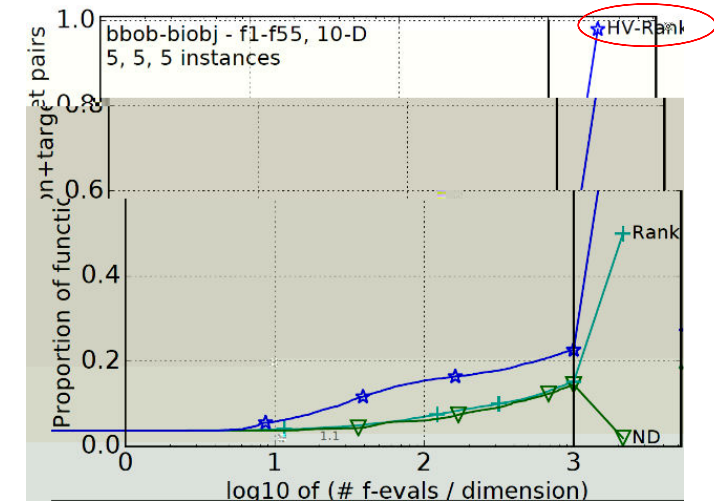
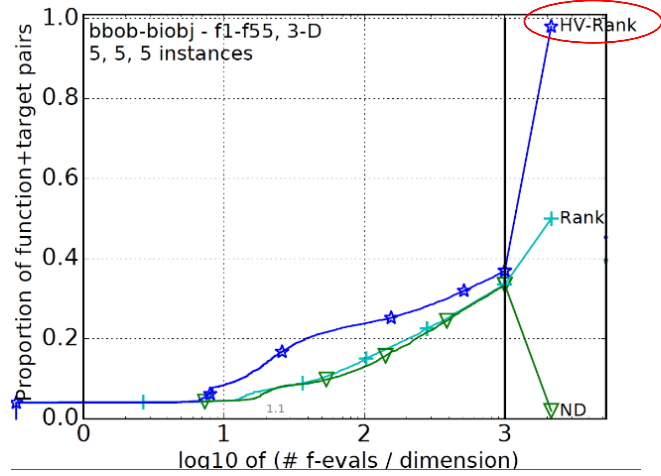
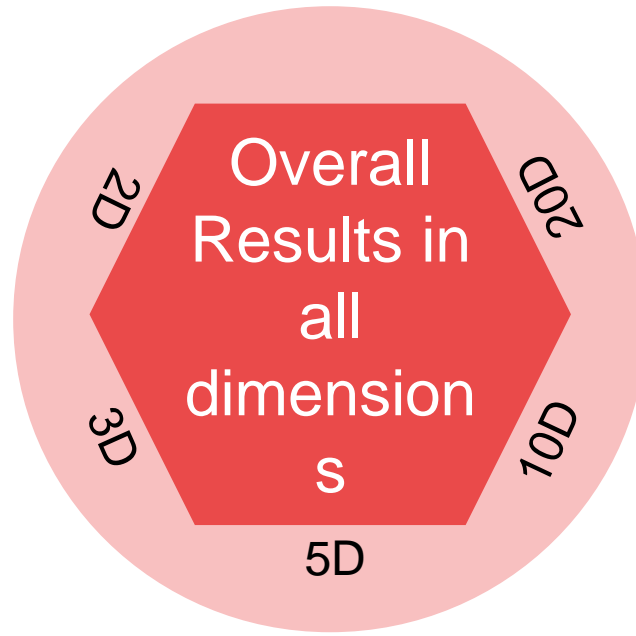
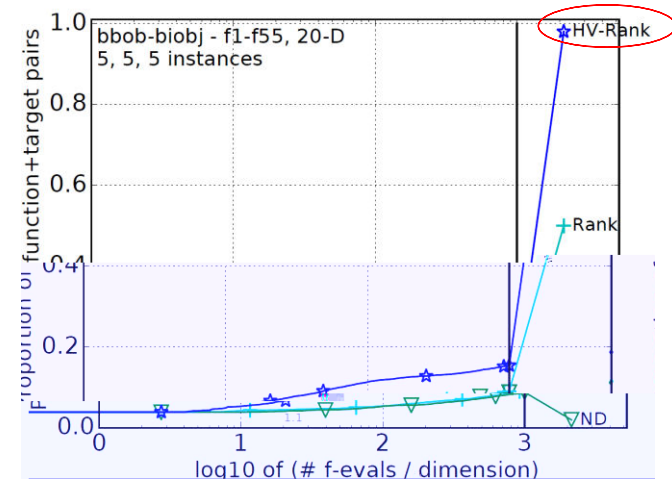
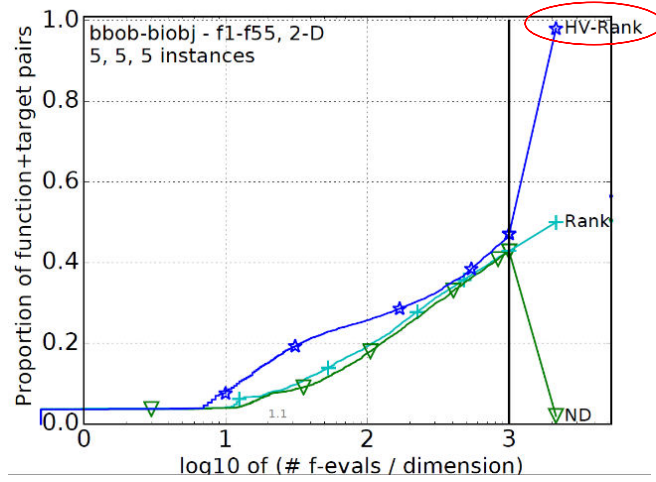
Need to  
search  
unexplored  
areas

ND: 11 points, 7 non-dominated  
Rank: 3 points, 1 non-dominated

Since, we want to explore unexplored areas not covered by non-dominated solutions, Rank is a better choice.







# Main Findings

In high dimensions, results start to form a plateau

Excessive sampling due to partitioning strategy

1 potentially optimal point  
=  $2 * 20$  sampled points in a  
 $20D$  problem

MO-DIRECT-hv  
outperforms  
ND and Rank  
strategies

MO-DIRECT-hv performs better in non multi-modal problems

Validates and reflects the proposed idea of MO-DIRECT-hv getting stuck in local minima

# Future Works

New Partitioning Strategies to reduce sampled points at high dimensions

Tuning of parameters to determine if the algorithm is stuck in local optima

Testing MO-DIRECT-hv on multiobjective benchmark problems