

# Comparison Tables: CEC BBOB 2015 Testbed in 2-D (Expensive Setting)

The BBOBies

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## Abstract

This document provides tabular results of the special session on Black-Box Optimization Benchmarking at CEC 2015 with a focus on benchmarking black-box algorithms for small function evaluation budgets (“expensive setting”), see <http://coco.gforge.inria.fr/doku.php?id=cec-bbob-2015>. Overall, eight algorithms have been tested on 24 benchmark functions in dimensions between 2 and 20. A description of the used objective functions can be found in [6, 4]. The experimental set-up is described in [5].

The performance measure provided in the following tables is the expected number of objective function evaluations to reach a given target function value (ERT, expected running time), divided by the respective value for the best algorithm in BBOB-2009 (see [1]) if an algorithm from BBOB-2009 reached the given target function value. The ERT value is given otherwise ( $\text{ERT}_{\text{best}}$  is noted as infinite). See [5] for details on how ERT is obtained. Bold entries in the table correspond to values below 3 or the top-three best values. Table 1 gives an overview on all algorithms submitted to the noise-free testbed at CEC 2015.

Table 1: Names and references of all algorithms submitted for the noise-free testbed

algorithm name	short	paper	reference
MATSuMoTo		Comparison of the MATSuMoTo Library for Expensive Optimization on the Noiseless Black-Box Optimization Benchmarking Testbed	[2]
R-DE-10e2		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
R-DE-10e5		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
R-SHADE-10e2		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
R-SHADE-10e5		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
RL-SHADE-10e2		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
RL-SHADE-10e5		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
SOO		Simultaneous Optimistic Optimization on the Noiseless BBOB Testbed	[3]

Table 2: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_1$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

↻

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f1</b>	<i>1.6e+1</i> :1.2	<i>4.0e+0</i> :2.6	<i>2.5e-2</i> :6.2	<i>1.0e-8</i> :6.2	<i>1.0e-8</i> :6.2	15/15
MATSUMOTO	<b>1.7</b> (2)	<b>1.8</b> (1)	<b>2.8</b> (0.8) <sup>*2</sup>	∞	∞ <i>100</i>	0/15
R-DE-10e2-	<b>2.7</b> (1)	<b>2.3</b> (3)	10(5)	∞	∞ <i>200</i>	0/15
R-DE-10e5-	<b>2.3</b> (1)	<b>3.0</b> (4)	66(222)	102(6)	102(150)	15/15
RL-SHADE-1	<b>1.8</b> (4)	<b>2.1</b> (0.8)	11(4)	240(210)	240(330)	2/15
RL-SHADE-1	<b>2.1</b> (2)	3.3(4)	30(17)	225(20)	225(26)	15/15
R-SHADE-10	<b>2.7</b> (2)	<b>2.8</b> (2)	13(5)	∞	∞ <i>200</i>	0/15
R-SHADE-10	<b>1.9</b> (1)	<b>2.9</b> (1)	11(4)	<b>45</b> (6)	<b>45</b> (7)	15/15
SOO-Derbel	<b>1.1</b> (0.2)	<b>1.2</b> (0.8)	<b>5.6</b> (2)	<b>52</b> (7)	<b>52</b> (3)	15/15

Table 3: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_2$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f</i><sub>2</sub></b>	<i>1.0e+7</i> :1.4	<i>1.6e+6</i> :2.7	<i>1.0e+5</i> :6.1	<i>6.3e-1</i> :20	<i>1.0e-8</i> :30	15/15
MATSUMOTO	<b>1.5</b> <sup>(0.7)</sup>	<b>1.6</b> <sup>(0.5)</sup>	<b>1.3</b> <sup>(1)</sup>	$\infty$	$\infty$ <i>100</i>	0/15
R-DE-10e2-	<b>1.6</b> <sup>(1)</sup>	<b>1.9</b> <sup>(2)</sup>	<b>2.2</b> <sup>(2)</sup>	<b>7.3</b> <sup>(3)</sup>	$\infty$ <i>200</i>	0/15
R-DE-10e5-	<b>1</b> <sup>(0.5)</sup>	<b>1.2</b> <sup>(2)</sup>	<b>2.1</b> <sup>(2)</sup>	13 <sup>(46)</sup>	<b>24</b> <sup>(3)</sup>	15/15
RL-SHADE-1	<b>1.1</b> <sup>(1)</sup>	<b>1.0</b> <sup>(0.5)</sup>	3.2 <sup>(2)</sup>	<b>6.4</b> <sup>(2)</sup>	$\infty$ <i>200</i>	0/15
RL-SHADE-1	<b>0.90</b> <sup>(0.4)</sup>	<b>1.5</b> <sup>(0.9)</sup>	<b>2.8</b> <sup>(3)</sup>	27 <sup>(13)</sup>	73 <sup>(4)</sup>	15/15
R-SHADE-10	<b>1.1</b> <sup>(0.9)</sup>	<b>1.1</b> <sup>(0.7)</sup>	5.0 <sup>(3)</sup>	12 <sup>(6)</sup>	$\infty$ <i>200</i>	0/15
R-SHADE-10	<b>1.1</b> <sup>(1)</sup>	<b>2.3</b> <sup>(2)</sup>	<b>2.2</b> <sup>(3)</sup>	8.7 <sup>(1)</sup>	<b>15</b> <sup>(2)</sup>	15/15
SOO-Derbel	<b>1.1</b> <sup>(0.4)</sup>	<b>1.2</b> <sup>(1)</sup>	<b>2.6</b> <sup>(1)</sup>	8.9 <sup>(2)</sup>	27 <sup>(4)</sup>	15/15

Table 4: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_3$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f3</b>	<i>1.0e+2:1.4</i>	<i>4.0e+1:4.1</i>	<i>2.5e+1:6.6</i>	<i>6.3e+0:26</i>	<i>2.5e+0:112</i>	15/15
MATSUMOTO	<b>1.6</b> <sup>(0.7)</sup>	<b>1.4</b> <sup>(1)</sup>	<b>1.5</b> <sup>(0.9)</sup>	<b>2.7</b> <sup>(2)</sup>	<b>1.4</b> <sup>(0.5)</sup>	8/15
R-DE-10e2-	<b>1.3</b> <sup>(1)</sup>	<b>1.3</b> <sup>(1)</sup>	<b>1.7</b> <sup>(1)</sup>	<b>2.2</b> <sup>(2)</sup>	<b>0.87</b> <sup>(0.6)</sup>	15/15
R-DE-10e5-	<b>1.0</b> <sup>(0.7)</sup>	<b>2.1</b> <sup>(2)</sup>	<b>2.3</b> <sup>(1)</sup>	<b>2.1</b> <sup>(2)</sup>	<b>2.2</b> <sup>(2)</sup>	15/15
RL-SHADE-1	<b>1.4</b> <sup>(1)</sup>	<b>1.5</b> <sup>(1)</sup>	<b>2.3</b> <sup>(2)</sup>	<b>1.8</b> <sup>(0.7)</sup>	<b>1.0</b> <sup>(0.2)</sup>	13/15
RL-SHADE-1	<b>1.8</b> <sup>(2)</sup>	<b>2.3</b> <sup>(2)</sup>	3.8 <sup>(3)</sup>	4.9 <sup>(4)</sup>	<b>2.8</b> <sup>(1.0)</sup>	15/15
R-SHADE-10	<b>1.2</b> <sup>(0.5)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>1.9</b> <sup>(1)</sup>	<b>0.80</b> <sup>(0.6)</sup>	15/15
R-SHADE-10	<b>1.5</b> <sup>(1)</sup>	<b>2.3</b> <sup>(2)</sup>	<b>2.8</b> <sup>(3)</sup>	5.4 <sup>(9)</sup>	<b>2.0</b> <sup>(2)</sup>	15/15
SOO-Derbel	<b>0.86</b> <sup>(0.2)</sup>	<b>1.3</b> <sup>(0.9)</sup>	<b>1.7</b> <sup>(2)</sup>	<b>1.9</b> <sup>(1)</sup>	<b>0.90</b> <sup>(0.5)</sup>	15/15

Table 5: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_4$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><math>f_4</math></b>	<i>6.3e+1</i> :2.4	<i>4.0e+1</i> :5.2	<i>2.5e+1</i> :8.5	<i>1.0e+1</i> :22	<i>2.5e+0</i> :120	5/5
MATSUMOTO	<b>1.9</b> (1)	<b>1.5</b> (0.9)	<b>1.8</b> (1)	<b>1.9</b> (1)	<b>2.2</b> (2)	5/15
R-DE-10e2-	<b>1.6</b> (0.9)	<b>1.7</b> (1)	<b>2.0</b> (2)	<b>1.9</b> (2)	<b>1.2</b> (0.4)	13/15
R-DE-10e5-	<b>2.5</b> (2)	<b>2.8</b> (2)	5.5(2)	5.1(11)	<b>1.7</b> (0.3)	15/15
RL-SHADE-1	<b>1.4</b> (1)	<b>1.6</b> (1)	<b>1.5</b> (2)	<b>1.8</b> (0.9)	<b>0.81</b> (0.2)	14/15
RL-SHADE-1	<b>2.1</b> (2)	<b>1.5</b> (0.9)	4.1(3)	5.2(1)	<b>2.9</b> (1)	15/15
R-SHADE-10	<b>1.8</b> (1)	<b>2.4</b> (2)	<b>1.9</b> (1)	<b>2.0</b> (2)	<b>1.1</b> (0.5)	13/15
R-SHADE-10	<b>1.9</b> (2)	<b>1.6</b> (2)	<b>1.8</b> (1)	<b>2.4</b> (0.6)	<b>3.0</b> (6)	15/15
SOO-Derbel	<b>0.83</b> (0.4)	<b>1.2</b> (0.6)	<b>1.5</b> (1)	<b>1.5</b> (0.8)	<b>1.2</b> (0.4)	15/15

Table 6: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_5$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f5</b>	<i>4.0e+1</i> :1.4	<i>1.6e+1</i> :3.5	<i>1.0e-8</i> :4.4	<i>1.0e-8</i> :4.4	<i>1.0e-8</i> :4.4	15/15
MATSUMOTO	<b>1.5</b> (1)	<b>1.2</b> (0.9)	<b>2.5</b> (0.9) <sup>*4</sup>	<b>2.5</b> (1) <sup>*4</sup>	<b>2.5</b> (0.6) <sup>*4</sup>	15/15
R-DE-10e2-	<b>1.9</b> (0.7)	3.1(3)	$\infty$	$\infty$	$\infty$ 200	0/15
R-DE-10e5-	<b>1.5</b> (1)	<b>2.7</b> (2)	319(272)	319(367)	319(295)	15/15
RL-SHADE-1	<b>2.0</b> (1)	3.0(2)	320(474)	320(253)	320(276)	2/15
RL-SHADE-1	<b>1.3</b> (0.4)	<b>1.7</b> (3)	349(24)	349(33)	349(14)	15/15
R-SHADE-10	<b>2.0</b> (1)	<b>2.9</b> (3)	$\infty$	$\infty$	$\infty$ 200	0/15
R-SHADE-10	<b>1.9</b> (1)	<b>2.0</b> (0.9)	<b>133</b> (49)	<b>133</b> (52)	<b>133</b> (22)	15/15
SOO-Derbel	<b>1.8</b> (0.4)	<b>2.5</b> (0.1)	338(0.1)	338(0.1)	338(0.1)	15/15

Table 7: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_6$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

∞

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f6</b>	<i>6.3e+4</i> :1.4	<i>1.0e+2</i> :2.8	<i>1.6e+1</i> :10	<i>1.0e+0</i> :23	<i>2.5e-6</i> :103	15/15
MATSUMOTO	<b>1.3</b> <sup>(0.9)</sup>	<b>1.7</b> <sup>(1)</sup>	<b>1.3</b> <sup>(0.9)</sup>	14 <sup>(9)</sup>	∞ <i>100</i>	0/15
R-DE-10e2-	<b>1.4</b> <sup>(0.9)</sup>	<b>2.1</b> <sup>(2)</sup>	<b>1.4</b> <sup>(2)</sup>	<b>2.3</b> <sup>(0.8)</sup>	∞ <i>200</i>	0/15
R-DE-10e5-	<b>1.5</b> <sup>(0.9)</sup>	<b>1.8</b> <sup>(2)</sup>	12 <sup>(4)</sup>	20 <sup>(66)</sup>	18 <sup>(21)</sup>	15/15
RL-SHADE-1	<b>1.2</b> <sup>(0.4)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>2.0</b> <sup>(2)</sup>	4.2 <sup>(2)</sup>	∞ <i>200</i>	0/15
RL-SHADE-1	<b>1.4</b> <sup>(2)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>2.4</b> <sup>(2)</sup>	7.1 <sup>(5)</sup>	<b>16</b> <sup>(4)</sup>	15/15
R-SHADE-10	<b>1.0</b> <sup>(0.5)</sup>	<b>1.3</b> <sup>(1)</sup>	<b>1.2</b> <sup>(0.8)</sup>	4.2 <sup>(2)</sup>	∞ <i>200</i>	0/15
R-SHADE-10	<b>1.4</b> <sup>(2)</sup>	<b>2.3</b> <sup>(3)</sup>	<b>1.7</b> <sup>(3)</sup>	<b>3.2</b> <sup>(2)</sup>	<b>4.9</b> <sup>(3)</sup>	15/15
SOO-Derbel	<b>1</b> <sup>(1)</sup>	<b>0.95</b> <sup>(0.4)</sup>	<b>1.2</b> <sup>(2)</sup>	7.6 <sup>(7)</sup>	1.4e4 <sup>(2e4)</sup>	2/15



Table 8: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_7$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f7</i></b>	<i>4.0e+2</i> :1.6	<i>1.0e+1</i> :3.2	<i>2.5e+0</i> :14	<i>1.6e+0</i> :21	<i>1.6e-2</i> :188	15/15
MATSUMOTO	<b>1.4</b> (1)	<b>2.8</b> (2)	<b>1.2</b> (0.9)	<b>1.1</b> (0.3)	<b>0.55</b> (0.4)	10/15
R-DE-10e2-	<b>1.5</b> (0.8)	3.1(2)	<b>1.9</b> (1)	<b>2.0</b> (2)	<b>1.0</b> (1)	11/15
R-DE-10e5-	<b>0.87</b> (0.6)	<b>2.2</b> (2)	<b>1.5</b> (2)	<b>2.6</b> (5)	<b>1.6</b> (0.8)	15/15
RL-SHADE-1	<b>1</b> (1)	5.9(4)	<b>2.5</b> (2)	<b>2.8</b> (4)	<b>2.4</b> (2)	6/15
RL-SHADE-1	<b>0.96</b> (0.5)	4.9(7)	<b>2.9</b> (4)	<b>3.0</b> (4)	<b>2.3</b> (1)	15/15
R-SHADE-10	<b>1.0</b> (0.3)	<b>2.9</b> (1)	<b>1.4</b> (0.8)	<b>2.0</b> (2)	4.9(4)	3/15
R-SHADE-10	<b>0.75</b> (0.3)	<b>2.6</b> (3)	<b>1.4</b> (2)	<b>1.4</b> (1)	<b>0.81</b> (0.6)	15/15
SOO-Derbel	<b>1.2</b> (1)	<b>1.8</b> (2)	<b>1.0</b> (1.0)	<b>1.3</b> (0.7)	<b>0.76</b> (0.4)	15/15

Table 9: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_8$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f8</b>	<i>2.5e+3</i> :1.2	<i>1.0e+2</i> :3.2	<i>6.3e+0</i> :7.0	<i>1.6e-1</i> :27	<i>1.6e-6</i> :100	15/15
MATSUMOTO	<b>1.7</b> (1)	<b>2.1</b> (1)	<b>2.6</b> (1)	7.7(11)	$\infty$ 100	0/15
R-DE-10e2-	<b>1.5</b> (0.8)	4.2(4)	4.1(4)	16(5)	$\infty$ 200	0/15
R-DE-10e5-	<b>2.0</b> (0.6)	<b>2.5</b> (0.8)	<b>2.8</b> (3)	16(8)	<b>12</b> (6)	15/15
RL-SHADE-1	<b>1.8</b> (1)	<b>2.5</b> (3)	3.8(4)	6.3(2)	$\infty$ 200	0/15
RL-SHADE-1	<b>2.1</b> (0.4)	<b>2.6</b> (1)	7.8(8)	12(5)	23(3)	15/15
R-SHADE-10	<b>2.3</b> (1)	<b>3.0</b> (3)	7.6(8)	15(34)	$\infty$ 200	0/15
R-SHADE-10	<b>2.0</b> (2)	<b>2.7</b> (2)	3.4(2)	<b>4.6</b> (5)	<b>5.3</b> (5)	15/15
SOO-Derbel	<b>1.7</b> (0.4)	<b>1.4</b> (1)	<b>1.9</b> (1)	<b>4.4</b> (3)	21(13)	15/15

Table 10: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_9$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f</i><sub>9</sub></b>	<i>6.3e+0:13</i>	<i>4.0e+0:15</i>	<i>2.5e+0:15</i>	<i>2.5e-1:21</i>	<i>1.0e-8:94</i>	15/15
MATSUMOTO	<b>1.3</b> (1)	<b>1.4</b> (0.9)	<b>1.6</b> (1)	<b>4.3</b> (2)	$\infty$ <i>100</i>	0/15
R-DE-10e2-	<b>3.0</b> (6)	3.5(5)	4.9(1)	8.3(9)	$\infty$ <i>200</i>	0/15
R-DE-10e5-	3.8(3)	4.6(2)	6.0(5)	27(86)	<b>20</b> (13)	15/15
RL-SHADE-1	3.9(6)	4.0(3)	4.4(7)	10(8)	$\infty$ <i>200</i>	0/15
RL-SHADE-1	3.2(2)	3.4(4)	3.6(4)	15(8)	28(2)	15/15
R-SHADE-10	3.4(2)	3.7(3)	5.0(4)	16(15)	$\infty$ <i>200</i>	0/15
R-SHADE-10	<b>2.2</b> (2)	<b>2.0</b> (2)	<b>2.3</b> (3)	7.3(3)	<b>8.0</b> (12)	15/15
SOO-Derbel	<b>1.4</b> (0.8)	<b>1.6</b> (0.9)	<b>1.9</b> (0.8)	<b>2.8</b> (2)	25(16)	15/15

Table 11: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{10}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f10</b>	<i>1.6e+6:2.0</i>	<i>4.0e+5:3.2</i>	<i>6.3e+2:8.8</i>	<i>1.0e+1:30</i>	<i>2.5e-8:101</i>	15/15
MATSUMOTO	<b>2.3</b> (2)	<b>2.4</b> (2)	<b>2.9</b> (3)	9.1(11)	$\infty$ 100	0/15
R-DE-10e2-	<b>2.2</b> (1)	<b>2.0</b> (1)	6.8(6)	<b>7.3</b> (5)	$\infty$ 200	0/15
R-DE-10e5-	<b>2.8</b> (2)	<b>2.8</b> (2)	6.9(6)	27(30)	74(51)	15/15
RL-SHADE-1	<b>1.7</b> (2)	<b>1.7</b> (1)	7.1(6)	18(24)	$\infty$ 200	0/15
RL-SHADE-1	<b>2.0</b> (2)	<b>2.2</b> (1)	14(8)	15(8)	<b>26</b> (2)	15/15
R-SHADE-10	<b>2.2</b> (2)	<b>1.6</b> (1)	11(10)	13(11)	$\infty$ 200	0/15
R-SHADE-10	<b>2.2</b> (2)	<b>2.4</b> (2)	5.0(3)	<b>3.2</b> (0.9)	<b>4.4</b> (1)	15/15
SOO-Derbel	<b>1.3</b> (0.8)	<b>1.1</b> (0.9)	<b>3.9</b> (3)	<b>3.3</b> (2)	494(196)	15/15

Table 12: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_{11}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f11</b>	<i>1.0e+7</i> :1.1	<i>1.6e+6</i> :3.2	<i>1.0e+4</i> :6.6	<i>4.0e+1</i> :23	<i>4.0e-8</i> :100	15/15
MATSUMOTO	<b>1.6</b> <sup>(0.9)</sup>	<b>1.2</b> <sup>(1)</sup>	<b>2.4</b> <sup>(1)</sup>	<b>2.8</b> <sup>(2)</sup>	$\infty$ 100	0/15
R-DE-10e2-	<b>1.8</b> <sup>(0.5)</sup>	<b>1.1</b> <sup>(0.8)</sup>	4.2 <sup>(4)</sup>	5.1 <sup>(3)</sup>	$\infty$ 200	0/15
R-DE-10e5-	<b>1.9</b> <sup>(2)</sup>	<b>1.8</b> <sup>(1)</sup>	<b>3.0</b> <sup>(2)</sup>	10 <sup>(20)</sup>	94 <sup>(103)</sup>	15/15
RL-SHADE-1	<b>2.0</b> <sup>(0.7)</sup>	<b>1.6</b> <sup>(2)</sup>	4.4 <sup>(4)</sup>	5.7 <sup>(5)</sup>	$\infty$ 200	0/15
RL-SHADE-1	<b>2.1</b> <sup>(3)</sup>	<b>1.9</b> <sup>(2)</sup>	8.6 <sup>(12)</sup>	12 <sup>(5)</sup>	<b>25</b> <sup>(3)</sup>	15/15
R-SHADE-10	<b>1.8</b> <sup>(0.9)</sup>	<b>1.5</b> <sup>(1)</sup>	4.1 <sup>(2)</sup>	9.0 <sup>(7)</sup>	$\infty$ 200	0/15
R-SHADE-10	<b>1.8</b> <sup>(0.5)</sup>	<b>2.2</b> <sup>(1)</sup>	4.4 <sup>(4)</sup>	4.1 <sup>(5)</sup>	<b>5.0</b> <sup>(2)</sup>	15/15
SOO-Derbel	<b>1.2</b> <sup>(0.5)</sup>	<b>0.94</b> <sup>(0.8)</sup>	<b>2.7</b> <sup>(1)</sup>	<b>2.9</b> <sup>(0.9)</sup>	481 <sup>(197)</sup>	15/15

Table 13: 02-D, running time excess  $ERT/ERT_{\text{best } 2009}$  on  $f_{12}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $ERT_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f12</b>	<i>2.5e+8:1.3</i>	<i>6.3e+6:2.7</i>	<i>6.3e+5:6.3</i>	<i>4.0e+1:21</i>	<i>1.6e-3:101</i>	15/15
MATSUMOTO	<b>1.1</b> <sup>(0.8)</sup>	<b>0.93</b> <sup>(0.7)</sup>	<b>0.77</b> <sup>(0.4)</sup>	4.6 <sup>(3)</sup>	$\infty$ <i>100</i>	0/15
R-DE-10e2-	<b>0.85</b> <sup>(0.2)</sup>	<b>0.80</b> <sup>(0.5)</sup>	<b>1.0</b> <sup>(1.0)</sup>	5.3 <sup>(5)</sup>	29 <sup>(22)</sup>	1/15
R-DE-10e5-	<b>0.95</b> <sup>(0)</sup>	<b>0.98</b> <sup>(0.5)</sup>	<b>0.98</b> <sup>(0.5)</sup>	8.0 <sup>(1.0)</sup>	37 <sup>(55)</sup>	15/15
RL-SHADE-1	<b>1.6</b> <sup>(2)</sup>	<b>1.5</b> <sup>(2)</sup>	<b>1.1</b> <sup>(0.6)</sup>	<b>3.9</b> <sup>(2)</sup>	29 <sup>(27)</sup>	1/15
RL-SHADE-1	<b>1</b> <sup>(1)</sup>	<b>0.76</b> <sup>(0.7)</sup>	<b>1.4</b> <sup>(1)</sup>	13 <sup>(5)</sup>	<b>26</b> <sup>(9)</sup>	15/15
R-SHADE-10	<b>1.1</b> <sup>(0.6)</sup>	<b>1.7</b> <sup>(1)</sup>	<b>1.4</b> <sup>(1.0)</sup>	7.8 <sup>(3)</sup>	$\infty$ <i>200</i>	0/15
R-SHADE-10	<b>0.80</b> <sup>(0.2)</sup>	<b>0.90</b> <sup>(0.7)</sup>	<b>1.6</b> <sup>(0.8)</sup>	4.7 <sup>(4)</sup>	<b>15</b> <sup>(15)</sup>	15/15
SOO-Derbel	<b>0.95</b> <sup>(0.8)</sup>	<b>0.73</b> <sup>(0.7)</sup>	<b>0.59</b> <sup>(0.5)</sup>	<b>4.3</b> <sup>(2)</sup>	33 <sup>(20)</sup>	15/15

Table 14: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{13}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f13</b>	<i>4.0e+2</i> :1.6	<i>2.5e+2</i> :3.1	<i>6.3e+1</i> :8.7	<i>1.0e+1</i> :23	<i>4.0e-6</i> :100	15/15
MATSUMOTO	<b>1.6</b> <sup>(0.9)</sup>	<b>1.5</b> <sup>(0.6)</sup>	<b>1.4</b> <sup>(0.5)</sup>	<b>1.6</b> <sup>(0.4)</sup>	$\infty$ 100	0/15
R-DE-10e2-	<b>1.5</b> <sup>(2)</sup>	<b>1.6</b> <sup>(4)</sup>	<b>2.1</b> <sup>(2)</sup>	4.0 <sup>(0.7)</sup>	$\infty$ 200	0/15
R-DE-10e5-	<b>1.8</b> <sup>(1)</sup>	<b>1.5</b> <sup>(1)</sup>	<b>2.4</b> <sup>(5)</sup>	4.5 <sup>(9)</sup>	100 <sup>(94)</sup>	15/15
RL-SHADE-1	<b>1.4</b> <sup>(2)</sup>	<b>1.5</b> <sup>(0.6)</sup>	<b>2.4</b> <sup>(2)</sup>	3.9 <sup>(3)</sup>	$\infty$ 200	0/15
RL-SHADE-1	<b>1.2</b> <sup>(0.3)</sup>	<b>0.68</b> <sup>(0.5)</sup>	<b>2.5</b> <sup>(4)</sup>	4.7 <sup>(4)</sup>	<b>25</b> <sup>(2)</sup>	15/15
R-SHADE-10	<b>2.3</b> <sup>(1)</sup>	<b>1.6</b> <sup>(1)</sup>	<b>3.0</b> <sup>(2)</sup>	5.8 <sup>(6)</sup>	$\infty$ 200	0/15
R-SHADE-10	<b>1.5</b> <sup>(1)</sup>	<b>1.3</b> <sup>(1)</sup>	<b>1.8</b> <sup>(1)</sup>	<b>2.6</b> <sup>(0.8)</sup>	<b>4.9</b> <sup>(0.4)</sup>	15/15
SOO-Derbel	<b>1.4</b> <sup>(3)</sup>	<b>1.1</b> <sup>(1)</sup>	<b>1.8</b> <sup>(1)</sup>	<b>2.8</b> <sup>(4)</sup>	153 <sup>(161)</sup>	15/15

Table 15: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{14}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f14</b>	<i>1.6e+1:1.4</i>	<i>2.5e+0:4.2</i>	<i>1.0e+0:7.4</i>	<i>2.5e-2:21</i>	<i>1.0e-8:101</i>	15/15
MATSUMOTO	<b>1.0</b> (0.7)	<b>1.4</b> (1)	<b>1.5</b> (1)	<b>1.7</b> (0.5) <sup>*2</sup>	$\infty$ 100	0/15
R-DE-10e2-	<b>1.3</b> (0.7)	<b>2.0</b> (2)	3.1(2)	4.2(1)	$\infty$ 200	0/15
R-DE-10e5-	<b>1.0</b> (0.7)	<b>1.1</b> (1)	<b>1.9</b> (0.8)	4.2(0.4)	94(129)	15/15
RL-SHADE-1	<b>1.2</b> (0.7)	<b>2.2</b> (3)	<b>2.7</b> (3)	4.1(2)	$\infty$ 200	0/15
RL-SHADE-1	<b>1.3</b> (0.4)	3.2(1)	4.3(4)	13(6)	<b>24</b> (2)	15/15
R-SHADE-10	<b>1.4</b> (1)	<b>2.0</b> (3)	3.3(3)	5.5(3)	$\infty$ 200	0/15
R-SHADE-10	<b>1.3</b> (0.7)	<b>2.8</b> (3)	3.8(4)	4.2(1)	<b>4.1</b> (0.4)	15/15
SOO-Derbel	<b>0.81</b> (0)	<b>1.6</b> (1)	<b>1.8</b> (1)	<b>3.2</b> (1)	43(19)	15/15



Table 16: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{15}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f15</b>	<i>1.6e+2:1.2</i>	<i>4.0e+1:4.7</i>	<i>2.5e+1:10</i>	<i>1.0e+1:37</i>	<i>2.5e+0:118</i>	5/5
MATSUMOTO	<b>1.6</b> (1)	<b>1.1</b> (0.7)	<b>0.79</b> (0.8)	<b>0.85</b> (0.6)	<b>1.0</b> (0.8)	9/15
R-DE-10e2-	<b>1.7</b> (0.8)	<b>1</b> (0.8)	<b>1.3</b> (1)	<b>1.2</b> (0.5)	<b>1.9</b> (2)	10/15
R-DE-10e5-	<b>1.8</b> (1)	<b>1.4</b> (0.9)	<b>1.2</b> (1)	<b>0.94</b> (1)	<b>2.1</b> (4)	15/15
RL-SHADE-1	<b>2.2</b> (1)	<b>1.6</b> (1)	<b>1.4</b> (1)	<b>1.8</b> (1)	<b>2.0</b> (2)	9/15
RL-SHADE-1	<b>1.8</b> (1)	<b>2.0</b> (4)	<b>1.4</b> (2)	<b>1.5</b> (2)	<b>2.7</b> (2)	15/15
R-SHADE-10	<b>1.5</b> (1)	<b>2.0</b> (2)	<b>1.4</b> (0.8)	<b>1.7</b> (1)	4.4(2)	5/15
R-SHADE-10	<b>1.3</b> (0.2)	<b>1.2</b> (1)	<b>1.7</b> (0.8)	<b>1.0</b> (0.8)	<b>1.4</b> (2)	15/15
SOO-Derbel	<b>1.1</b> (0.4)	<b>0.84</b> (1)	<b>0.74</b> (0.6)	<b>0.75</b> (0.4)	<b>1.1</b> (0.8)	15/15

Table 17: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{16}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f16</b>	<i>1.0e+2</i> :1.1	<i>2.5e+1</i> :3.9	<i>1.6e+1</i> :6.5	<i>4.0e+0</i> :31	<i>2.5e-1</i> :127	5/5
MATSUMOTO	<b>1.3</b> (1)	<b>1.7</b> (1)	<b>1.6</b> (2)	<b>1.1</b> (1)	<b>1.3</b> (2)	7/15
R-DE-10e2-	<b>1.4</b> (0.4)	<b>2.1</b> (2)	<b>1.8</b> (1)	<b>1.8</b> (2)	3.8(8)	5/15
R-DE-10e5-	<b>1.3</b> (0.9)	<b>1.3</b> (1)	<b>2.4</b> (3)	4.3(1)	<b>2.6</b> (3)	15/15
RL-SHADE-1	<b>1.1</b> (0)	<b>2.1</b> (2)	<b>1.8</b> (2)	<b>2.2</b> (1)	5.0(5)	4/15
RL-SHADE-1	<b>1.4</b> (0.9)	<b>2.4</b> (2)	3.3(3)	<b>2.2</b> (2)	4.5(2)	15/15
R-SHADE-10	<b>1.4</b> (0.7)	<b>2.2</b> (2)	<b>2.3</b> (3)	<b>1.7</b> (1)	4.0(5)	5/15
R-SHADE-10	<b>1.2</b> (0.4)	<b>2.3</b> (2)	<b>1.8</b> (2)	<b>1.9</b> (3)	<b>2.2</b> (1)	15/15
SOO-Derbel	<b>1.5</b> (0.4)	<b>2.3</b> (2)	<b>1.9</b> (1)	<b>1.0</b> (0.6)	<b>0.93</b> (1)	15/15

Table 18: 02-D, running time excess  $ERT/ERT_{\text{best } 2009}$  on  $f_{17}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $ERT_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f17</b>	<i>4.0e+1:1.2</i>	<i>1.0e+1:2.7</i>	<i>4.0e+0:10</i>	<i>2.5e+0:28</i>	<i>1.6e-1:119</i>	5/5
MATSUMOTO	<b>1.2</b> (0.6)	<b>1.5</b> (1)	<b>1.2</b> (0.3)	<b>0.52</b> (0.4)	<b>2.1</b> (3)	5/15
R-DE-10e2-	<b>1.7</b> (1)	3.2(2)	<b>1.6</b> (1)	<b>1.1</b> (1)	<b>1.3</b> (0.9)	14/15
R-DE-10e5-	<b>1.2</b> (0.4)	<b>1.8</b> (0.8)	<b>1.5</b> (2)	<b>0.85</b> (0.3)	<b>1.7</b> (2)	15/15
RL-SHADE-1	<b>1.2</b> (0.4)	<b>2.3</b> (3)	<b>1.8</b> (1.0)	<b>1.2</b> (2)	<b>1.6</b> (1)	11/15
RL-SHADE-1	<b>1.6</b> (2)	<b>1.9</b> (2)	<b>2.4</b> (3)	<b>1.4</b> (1)	3.4(1)	15/15
R-SHADE-10	<b>1.5</b> (1)	<b>2.8</b> (1)	<b>1.9</b> (2)	<b>1.0</b> (1)	4.6(6)	5/15
R-SHADE-10	<b>1.5</b> (1)	<b>2.3</b> (3)	<b>1.8</b> (2)	<b>1.1</b> (0.6)	<b>1.2</b> (0.5)	15/15
SOO-Derbel	<b>0.94</b> (0)	<b>1.8</b> (1)	<b>0.85</b> (0.5)	<b>0.48</b> (0.3)	<b>0.95</b> (0.8)	15/15

Table 19: 02-D, running time excess  $ERT/ERT_{\text{best } 2009}$  on  $f_{18}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $ERT_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f18</b>	<i>4.0e+2:1.2</i>	<i>1.0e+2:3.2</i>	<i>4.0e+1:7.2</i>	<i>6.3e+0:32</i>	<i>1.6e+0:104</i>	5/5
MATSUMOTO	<b>1.1</b> <sup>(0.4)</sup>	<b>0.83</b> <sup>(0.4)</sup>	<b>0.95</b> <sup>(0.4)</sup>	<b>0.86</b> <sup>(0.8)</sup>	<b>1.0</b> <sup>(0.6)</sup>	10/15
R-DE-10e2-	<b>1.3</b> <sup>(0.4)</sup>	<b>1.2</b> <sup>(0.5)</sup>	<b>1.5</b> <sup>(1)</sup>	<b>1.6</b> <sup>(0.7)</sup>	<b>1.0</b> <sup>(0.9)</sup>	14/15
R-DE-10e5-	<b>1.2</b> <sup>(0.2)</sup>	<b>1.2</b> <sup>(0.9)</sup>	<b>1.3</b> <sup>(1)</sup>	3.2 <sup>(0.8)</sup>	3.0 <sup>(7)</sup>	15/15
RL-SHADE-1	<b>1.2</b> <sup>(0.4)</sup>	<b>1.3</b> <sup>(1)</sup>	<b>2.1</b> <sup>(2)</sup>	<b>1.2</b> <sup>(1)</sup>	<b>1.6</b> <sup>(1)</sup>	10/15
RL-SHADE-1	<b>1.4</b> <sup>(0.6)</sup>	<b>1.6</b> <sup>(2)</sup>	<b>1.9</b> <sup>(2)</sup>	<b>2.6</b> <sup>(1)</sup>	<b>2.3</b> <sup>(1)</sup>	15/15
R-SHADE-10	<b>1.2</b> <sup>(0.4)</sup>	<b>0.98</b> <sup>(0.3)</sup>	<b>0.77</b> <sup>(1)</sup>	<b>1.2</b> <sup>(2)</sup>	<b>1.6</b> <sup>(0.8)</sup>	12/15
R-SHADE-10	<b>1.5</b> <sup>(0.6)</sup>	<b>1.8</b> <sup>(0.6)</sup>	<b>1.4</b> <sup>(1)</sup>	<b>1.4</b> <sup>(0.8)</sup>	<b>2.4</b> <sup>(0.4)</sup>	15/15
SOO-Derbel	<b>1.1</b> <sup>(0.4)</sup>	<b>1.6</b> <sup>(1)</sup>	<b>0.94</b> <sup>(0.9)</sup>	<b>0.73</b> <sup>(0.4)</sup>	<b>0.94</b> <sup>(0.5)</sup>	15/15

Table 20: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{19}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f19</b>	<i>1.6e-1:23</i>	<i>1.0e-1:26</i>	<i>6.3e-2:38</i>	<i>4.0e-2:40</i>	<i>1.0e-2:216</i>	15/15
MATSUMOTO	6.9(8)	9.3(4)	11(11)	11(12)	6.6(8)	1/15
R-DE-10e2-	4.3(2)	<b>4.2</b> (3)	5.5(2)	6.6(5)	6.7(5)	2/15
R-DE-10e5-	5.2(5)	5.6(6)	6.4(4)	12(14)	11(6)	15/15
RL-SHADE-1	5.4(6)	7.1(7)	6.9(7)	13(10)	<b>6.5</b> (7)	2/15
RL-SHADE-1	7.0(11)	7.6(8)	7.0(6)	14(14)	9.1(6)	15/15
R-SHADE-10	9.4(7)	8.4(11)	11(18)	24(33)	$\infty$ 200	0/15
R-SHADE-10	<b>2.9</b> (2)	<b>3.3</b> (1)	<b>3.2</b> (3)	<b>4.9</b> (5)	<b>5.2</b> (3)	15/15
SOO-Derbel	<b>4.1</b> (2)	5.0(4)	<b>3.6</b> (2)	<b>3.7</b> (2)	17(15)	15/15

Table 21: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{20}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f</i>20</b>	<i>4.0e+3</i> :1.9	<i>2.5e+2</i> :2.8	<i>4.0e+0</i> :6.3	<i>2.5e+0</i> :21	<i>6.3e-1</i> :139	15/15
MATSUMOTO	<b>1.5</b> <sup>(0.8)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>2.1</b> <sup>(1)</sup>	<b>1.8</b> <sup>(1)</sup>	3.4 <sup>(2)</sup>	3/15
R-DE-10e2-	<b>1</b> <sup>(0.5)</sup>	<b>1.7</b> <sup>(1)</sup>	<b>2.6</b> <sup>(3)</sup>	<b>1.8</b> <sup>(1)</sup>	<b>2.0</b> <sup>(3)</sup>	8/15
R-DE-10e5-	<b>1.2</b> <sup>(1)</sup>	<b>2.2</b> <sup>(2)</sup>	<b>1.9</b> <sup>(1)</sup>	<b>1.8</b> <sup>(1.0)</sup>	<b>2.4</b> <sup>(1)</sup>	15/15
RL-SHADE-1	<b>1.4</b> <sup>(0.9)</sup>	<b>2.7</b> <sup>(2)</sup>	3.1 <sup>(2)</sup>	<b>2.1</b> <sup>(1)</sup>	<b>2.7</b> <sup>(3)</sup>	7/15
RL-SHADE-1	<b>1.7</b> <sup>(1)</sup>	3.3 <sup>(3)</sup>	4.7 <sup>(3)</sup>	3.1 <sup>(3)</sup>	4.6 <sup>(2)</sup>	15/15
R-SHADE-10	<b>2.0</b> <sup>(5)</sup>	<b>2.6</b> <sup>(4)</sup>	4.0 <sup>(2)</sup>	<b>2.4</b> <sup>(1)</sup>	6.7 <sup>(5)</sup>	3/15
R-SHADE-10	<b>1.5</b> <sup>(2)</sup>	<b>2.5</b> <sup>(2)</sup>	3.2 <sup>(2)</sup>	<b>2.1</b> <sup>(1)</sup>	<b>2.3</b> <sup>(1)</sup>	15/15
SOO-Derbel	<b>1.4</b> <sup>(0.3)</sup>	5.1 <sup>(0.2)</sup>	<b>3.0</b> <sup>(0.1)</sup>	<b>1.3</b> <sup>(0.0)</sup>	<b>1.2</b> <sup>(4e-3)</sup>	15/15

Table 22: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_{21}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f21</i></b>	<i>1.0e+1:1.7</i>	<i>6.3e+0:2.6</i>	<i>2.5e+0:7.9</i>	<i>1.6e+0:30</i>	<i>4.0e-1:105</i>	15/15
MATSUMOTO	<b>1.1</b> (1)	<b>1.8</b> (2)	<b>1.7</b> (2)	<b>0.92</b> (1)	<b>0.64</b> (1)	12/15
R-DE-10e2-	<b>1.6</b> (2)	<b>2.6</b> (10)	<b>1.7</b> (4)	<b>2.1</b> (2)	<b>2.7</b> (2)	7/15
R-DE-10e5-	<b>1.3</b> (0.6)	<b>1.3</b> (1)	<b>1.5</b> (0.4)	6.9(2)	5.3(7)	15/15
RL-SHADE-1	<b>1.1</b> (0.4)	<b>1.5</b> (1)	<b>1.2</b> (0.8)	<b>0.92</b> (0.2)	<b>1.1</b> (0.5)	11/15
RL-SHADE-1	<b>1.6</b> (1)	<b>2.2</b> (2)	<b>1.3</b> (2)	<b>1.1</b> (1)	<b>1.2</b> (2)	15/15
R-SHADE-10	<b>1.5</b> (2)	<b>1.7</b> (2)	<b>1.4</b> (0.9)	<b>1.6</b> (1)	<b>1.1</b> (1)	12/15
R-SHADE-10	<b>1.3</b> (0.3)	<b>1.9</b> (3)	<b>2.0</b> (2)	<b>1.2</b> (0.7)	3.3(6)	15/15
SOO-Derbel	<b>0.88</b> (0.3)	<b>1.5</b> (0.7)	<b>1.3</b> (0.9)	<b>0.93</b> (0.8)	<b>0.53</b> (0.6)	15/15

Table 23: 02-D, running time excess  $ERT/ERT_{\text{best } 2009}$  on  $f_{22}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $ERT_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f22</b>	<i>4.0e+1:1.3</i>	<i>1.6e+1:3.2</i>	<i>6.3e+0:9.3</i>	<i>1.6e+0:25</i>	<i>1.0e-1:168</i>	15/15
MATSUMOTO	<b>1.7</b> <sup>(0.8)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>1.2</b> <sup>(1.0)</sup>	<b>1.1</b> <sup>(0.8)</sup>	<b>0.90</b> <sup>(0.9)</sup>	8/15
R-DE-10e2-	<b>1.9</b> <sup>(2)</sup>	<b>1.7</b> <sup>(2)</sup>	<b>1.5</b> <sup>(0.6)</sup>	<b>1.8</b> <sup>(4)</sup>	<b>0.64</b> <sup>(0.4)</sup>	13/15
R-DE-10e5-	<b>1.6</b> <sup>(1)</sup>	<b>1.5</b> <sup>(1)</sup>	<b>1.5</b> <sup>(2)</sup>	11 <sup>(2)</sup>	3.0 <sup>(6)</sup>	15/15
RL-SHADE-1	<b>1.5</b> <sup>(0.4)</sup>	<b>1.4</b> <sup>(0.9)</sup>	<b>1.4</b> <sup>(1.0)</sup>	<b>1.8</b> <sup>(3)</sup>	<b>0.90</b> <sup>(2)</sup>	10/15
RL-SHADE-1	<b>1.3</b> <sup>(0.4)</sup>	<b>0.58</b> <sup>(0.2)</sup>	<b>0.74</b> <sup>(1)</sup>	<b>2.0</b> <sup>(2)</sup>	17 <sup>(1)</sup>	15/15
R-SHADE-10	<b>1.2</b> <sup>(0.6)</sup>	<b>0.77</b> <sup>(0.7)</sup>	<b>1.3</b> <sup>(2)</sup>	<b>2.5</b> <sup>(3)</sup>	<b>1.5</b> <sup>(2)</sup>	8/15
R-SHADE-10	<b>1.3</b> <sup>(0.8)</sup>	<b>1.2</b> <sup>(0.8)</sup>	<b>0.81</b> <sup>(0.5)</sup>	4.2 <sup>(12)</sup>	<b>1.9</b> <sup>(4)</sup>	15/15
SOO-Derbel	<b>1.3</b> <sup>(0.4)</sup>	<b>1.2</b> <sup>(0.9)</sup>	<b>0.78</b> <sup>(0.8)</sup>	<b>0.89</b> <sup>(1)</sup>	<b>0.71</b> <sup>(0.7)</sup>	15/15



Table 24: 02-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{23}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f23</i></b>	<i>4.0e+1:1.5</i>	<i>2.5e+1:2.6</i>	<i>1.0e+1:7.8</i>	<i>4.0e+0:55</i>	<i>2.5e+0:103</i>	5/5
MATSUMOTO	<b>1.2</b> <sup>(0.7)</sup>	<b>1.4</b> <sup>(1)</sup>	<b>1.2</b> <sup>(2)</sup>	<b>1.4</b> <sup>(2)</sup>	<b>1.9</b> <sup>(2)</sup>	6/15
R-DE-10e2-	<b>1.7</b> <sup>(0.8)</sup>	<b>1.9</b> <sup>(1)</sup>	<b>1.8</b> <sup>(1.0)</sup>	<b>1.3</b> <sup>(1.0)</sup>	<b>1.6</b> <sup>(3)</sup>	10/15
R-DE-10e5-	<b>1.6</b> <sup>(2)</sup>	<b>1.8</b> <sup>(1)</sup>	<b>1.7</b> <sup>(2)</sup>	<b>2.1</b> <sup>(7)</sup>	4.5 <sup>(7)</sup>	15/15
RL-SHADE-1	<b>0.96</b> <sup>(0.7)</sup>	<b>1.5</b> <sup>(2)</sup>	<b>1.9</b> <sup>(2)</sup>	<b>2.0</b> <sup>(4)</sup>	3.5 <sup>(5)</sup>	6/15
RL-SHADE-1	<b>1.2</b> <sup>(1.0)</sup>	<b>1.7</b> <sup>(3)</sup>	<b>2.0</b> <sup>(1)</sup>	<b>2.3</b> <sup>(1)</sup>	<b>2.1</b> <sup>(1)</sup>	15/15
R-SHADE-10	<b>1.3</b> <sup>(0.5)</sup>	<b>1.4</b> <sup>(1)</sup>	<b>2.4</b> <sup>(3)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>3.0</b> <sup>(5)</sup>	7/15
R-SHADE-10	<b>1.3</b> <sup>(2)</sup>	<b>1.4</b> <sup>(0.9)</sup>	<b>2.3</b> <sup>(4)</sup>	<b>2.4</b> <sup>(3)</sup>	<b>2.1</b> <sup>(1)</sup>	15/15
SOO-Derbel	<b>1.4</b> <sup>(2)</sup>	<b>1.3</b> <sup>(0.8)</sup>	<b>2.6</b> <sup>(3)</sup>	<b>1.5</b> <sup>(1)</sup>	<b>1.6</b> <sup>(1)</sup>	15/15

Table 25: 02-D, running time excess  $ERT/ERT_{\text{best } 2009}$  on  $f_{24}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $ERT_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f</i>24</b>	<i>4.0e+1:1.1</i>	<i>2.5e+1:2.7</i>	<i>1.6e+1:7.7</i>	<i>6.3e+0:44</i>	<i>2.5e+0:275</i>	5/5
MATSUMOTO	<b>1.6</b> <sup>(0.9)</sup>	<b>1.7</b> <sup>(2)</sup>	<b>1.1</b> <sup>(0.8)</sup>	<b>1.2</b> <sup>(2)</sup>	$\infty$ <i>100</i>	0/15
R-DE-10e2-	<b>1.5</b> <sup>(0.9)</sup>	<b>1.3</b> <sup>(0.9)</sup>	<b>0.77</b> <sup>(0.7)</sup>	<b>0.96</b> <sup>(0.6)</sup>	<b>2.4</b> <sup>(3)</sup>	4/15
R-DE-10e5-	<b>1.5</b> <sup>(0.9)</sup>	<b>1.7</b> <sup>(1)</sup>	<b>1.0</b> <sup>(1)</sup>	<b>1.5</b> <sup>(1)</sup>	8.3 <sup>(7)</sup>	15/15
RL-SHADE-1	<b>1.6</b> <sup>(1)</sup>	<b>1.6</b> <sup>(1)</sup>	<b>1.4</b> <sup>(2)</sup>	<b>1.7</b> <sup>(2)</sup>	5.0 <sup>(6)</sup>	2/15
RL-SHADE-1	<b>1.6</b> <sup>(0.9)</sup>	<b>1.4</b> <sup>(0.9)</sup>	<b>1.4</b> <sup>(2)</sup>	<b>1.3</b> <sup>(1)</sup>	<b>2.9</b> <sup>(2)</sup>	15/15
R-SHADE-10	<b>1.2</b> <sup>(0)</sup>	<b>2.5</b> <sup>(3)</sup>	<b>1.8</b> <sup>(3)</sup>	<b>1.0</b> <sup>(1)</sup>	11 <sup>(13)</sup>	1/15
R-SHADE-10	<b>1.5</b> <sup>(0.9)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>1.2</b> <sup>(1)</sup>	<b>0.75</b> <sup>(0.6)</sup>	<b>1.8</b> <sup>(1)</sup>	15/15
SOO-Derbel	<b>1.1</b> <sup>(0.2)</sup>	<b>1.4</b> <sup>(1)</sup>	<b>1.9</b> <sup>(0.8)</sup>	<b>0.88</b> <sup>(0.7)</sup>	<b>1.6</b> <sup>(3)</sup>	15/15

## References

- [1] Anne Auger, Steffen Finck, Nikolaus Hansen, and Raymond Ros. BBOB 2009: Comparison tables of all algorithms on all noiseless functions. Technical Report RT-0383, INRIA, April 2010.
- [2] Dimo Brockhoff. Comparison of the matsumoto library for expensive optimization on the noiseless black-box optimization benchmarking testbed. In *Proceedings of the IEEE Congress on Evolutionary Computation, CEC 2015, 25-28 May, Sendai, Japan, 2015*.
- [3] Bilel Derbel and Philippe Preux. Simultaneous optimistic optimization on the noiseless bbob testbed. In *Proceedings of the IEEE Congress on Evolutionary Computation, CEC 2015, 25-28 May, Sendai, Japan, 2015*.
- [4] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009. Updated February 2010.
- [5] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2012: Experimental setup. Technical report, INRIA, 2012.
- [6] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009. Updated February 2010.
- [7] Ryoji Tanabe and Alex Fukunaga. Parameter tuning for differential evolution for cheap, medium, and expensive computational budgets. In *Proceedings of the IEEE Congress on Evolutionary Computation, CEC 2015, 25-28 May, Sendai, Japan, 2015*.