

# Comparison Tables: CEC BBOB 2015 Testbed in 5-D (Expensive Setting)

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## Abstract

This document provides tabular results of the special session on Black-Box Optimization Benchmarking at CEC 2015 with a focus on benchmarking black-box algorithms for small function evaluation budgets (“expensive setting”), see <http://coco.gforge.inria.fr/doku.php?id=cec-bbob-2015>. Overall, eight algorithms have been tested on 24 benchmark functions in dimensions between 2 and 20. A description of the used objective functions can be found in [6, 4]. The experimental set-up is described in [5].

The performance measure provided in the following tables is the expected number of objective function evaluations to reach a given target function value (ERT, expected running time), divided by the respective value for the best algorithm in BBOB-2009 (see [1]) if an algorithm from BBOB-2009 reached the given target function value. The ERT value is given otherwise ( $\text{ERT}_{\text{best}}$  is noted as infinite). See [5] for details on how ERT is obtained. Bold entries in the table correspond to values below 3 or the top-three best values. Table 1 gives an overview on all algorithms submitted to the noise-free testbed at CEC 2015.

Table 1: Names and references of all algorithms submitted for the noise-free testbed

algorithm name	short	paper	reference
MATSuMoTo		Comparison of the MATSuMoTo Library for Expensive Optimization on the Noiseless Black-Box Optimization Benchmarking Testbed	[2]
R-DE-10e2		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
R-DE-10e5		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
R-SHADE-10e2		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
R-SHADE-10e5		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
RL-SHADE-10e2		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
RL-SHADE-10e5		Parameter Tuning for Differential Evolution for Cheap, Medium, and Expensive Computational Budgets	[7]
SOO		Simultaneous Optimistic Optimization on the Noiseless BBOB Testbed	[3]

Table 2: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_1$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f1</b>	<i>2.5e+1:4.8</i>	<i>1.6e+1:7.6</i>	<i>1.0e-8:12</i>	<i>1.0e-8:12</i>	<i>1.0e-8:12</i>	15/15
MATSUMOTO	<b>1.8</b> <sup>(1)</sup>	<b>1.7</b> <sup>(0.8)</sup>	$\infty$	$\infty$	$\infty$ <i>250</i>	0/15
R-DE-10e2-	<b>2.5</b> <sup>(4)</sup>	<b>2.6</b> <sup>(2)</sup>	$\infty$	$\infty$	$\infty$ <i>500</i>	0/15
R-DE-10e5-	<b>2.5</b> <sup>(2)</sup>	<b>2.8</b> <sup>(3)</sup>	<b>67</b> <sup>(13)</sup>	<b>67</b> <sup>(8)</sup>	<b>67</b> <sup>(14)</sup>	15/15
RL-SHADE-1	<b>1.7</b> <sup>(1)</sup>	<b>2.7</b> <sup>(1)</sup>	614 <sup>(553)</sup>	614 <sup>(430)</sup>	614 <sup>(932)</sup>	1/15
RL-SHADE-1	<b>2.8</b> <sup>(11)</sup>	3.8 <sup>(5)</sup>	632 <sup>(17)</sup>	632 <sup>(20)</sup>	632 <sup>(36)</sup>	15/15
R-SHADE-10	<b>2.8</b> <sup>(2)</sup>	<b>2.6</b> <sup>(1)</sup>	$\infty$	$\infty$	$\infty$ <i>500</i>	0/15
R-SHADE-10	<b>2.2</b> <sup>(2)</sup>	3.4 <sup>(2)</sup>	<b>110</b> <sup>(9)</sup>	<b>110</b> <sup>(9)</sup>	<b>110</b> <sup>(11)</sup>	15/15
SOO-Derbel	<b>0.99</b> <sup>(1)</sup>	<b>1.3</b> <sup>(0.8)</sup>	194 <sup>(12)</sup>	194 <sup>(14)</sup>	194 <sup>(18)</sup>	15/15

Table 3: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_2$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f2</b>	<i>1.6e+6:2.9</i>	<i>4.0e+5:11</i>	<i>4.0e+4:15</i>	<i>6.3e+2:58</i>	<i>1.0e-8:95</i>	15/15
MATSUMOTO	<b>1.6</b> (1.0)	<b>0.68</b> (0.4)	<b>4.0</b> (5)	$\infty$	$\infty$ <i>250</i>	0/15
R-DE-10e2-	<b>2.0</b> (4)	<b>1.2</b> (1)	<b>4.0</b> (2)	<b>2.8</b> (0.5)	$\infty$ <i>500</i>	0/15
R-DE-10e5-	<b>2.1</b> (1)	<b>1.6</b> (2)	4.4(4)	<b>2.9</b> (0.9)	<b>10</b> (0.6)	15/15
RL-SHADE-1	<b>2.5</b> (3)	<b>1.4</b> (0.6)	5.2(3)	<b>2.9</b> (0.7)	$\infty$ <i>500</i>	0/15
RL-SHADE-1	<b>1.8</b> (2)	<b>0.92</b> (3)	10(10)	22(5)	113(4)	15/15
R-SHADE-10	<b>2.9</b> (8)	<b>2.2</b> (3)	4.6(4)	3.7(0.6)	$\infty$ <i>500</i>	0/15
R-SHADE-10	<b>1.7</b> (0.7)	<b>0.85</b> (1)	4.2(6)	5.4(1)	<b>22</b> (2)	15/15
SOO-Derbel	6.6(11)	<b>2.5</b> (3)	6.7(5)	6.6(2)	872(2643)	13/15

Table 4: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_3$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f3</b>	<i>1.6e+2:4.1</i>	<i>1.0e+2:15</i>	<i>6.3e+1:23</i>	<i>2.5e+1:73</i>	<i>1.0e+1:716</i>	15/15
MATSUMOTO	<b>1.8</b> (2)	<b>1.2</b> (1)	<b>1.8</b> (1)	<b>2.7</b> (2)	<b>1.1</b> (1)	4/15
R-DE-10e2-	<b>2.5</b> (3)	<b>1.1</b> (1)	<b>1.5</b> (0.9)	<b>1.8</b> (1)	<b>0.41</b> (0.1)	15/15
R-DE-10e5-	<b>1.9</b> (3)	<b>1.5</b> (1)	<b>1.7</b> (1)	<b>1.5</b> (0.8)	<b>0.36</b> (0.1)	15/15
RL-SHADE-1	<b>2.9</b> (2)	<b>2.5</b> (1)	<b>2.5</b> (1)	<b>1.7</b> (0.4)	<b>0.33</b> (0.4)	14/15
RL-SHADE-1	<b>1.3</b> (1.0)	<b>1.9</b> (4)	7.8(2)	11(3)	3.1(1)	15/15
R-SHADE-10	4.6(3)	<b>2.3</b> (1)	<b>2.3</b> (1.0)	<b>1.6</b> (0.5)	<b>0.33</b> (0.0)	15/15
R-SHADE-10	<b>1.5</b> (2)	<b>1.1</b> (1)	<b>2.4</b> (0.9)	3.0(0.6)	<b>1.1</b> (0.5)	15/15
SOO-Derbel	<b>1.9</b> (1)	<b>1.2</b> (0.5)	<b>1.9</b> (1.0)	<b>2.4</b> (1)	<b>1.2</b> (2)	15/15

Table 5: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_4$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><math>f_4</math></b>	<i>2.5e+2</i> :2.6	<i>1.6e+2</i> :10	<i>1.0e+2</i> :19	<i>4.0e+1</i> :65	<i>1.6e+1</i> :434	15/15
MATSUMOTO	<b>2.6</b> (4)	<b>1.1</b> (0.7)	<b>2.9</b> (2)	3.0(1)	9.0(12)	1/15
R-DE-10e2-	<b>2.1</b> (2)	<b>1.5</b> (1)	<b>1.8</b> (0.7)	<b>1.9</b> (0.5)	<b>0.92</b> (0.4)	12/15
R-DE-10e5-	4.3(4)	<b>2.5</b> (2)	<b>2.7</b> (2)	<b>2.2</b> (1)	<b>1.3</b> (2)	15/15
RL-SHADE-1	<b>2.7</b> (0.8)	<b>2.0</b> (2)	<b>2.5</b> (2)	<b>1.9</b> (0.7)	<b>0.49</b> (0.2)	15/15
RL-SHADE-1	3.0(1)	<b>2.2</b> (1)	4.2(5)	10(3)	4.1(1)	15/15
R-SHADE-10	3.3(3)	<b>2.0</b> (2)	<b>2.0</b> (1)	<b>1.6</b> (0.6)	<b>0.54</b> (0.2)	15/15
R-SHADE-10	<b>2.4</b> (2)	<b>1.5</b> (0.7)	<b>2.1</b> (1)	<b>2.5</b> (0.6)	<b>1.4</b> (0.5)	15/15
SOO-Derbel	<b>0.69</b> (0)	<b>0.68</b> (0.7)	<b>1.3</b> (1)	<b>2.3</b> (1)	3.7(19)	15/15

Table 6: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_5$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f5</b>	<i>6.3e+1</i> :4.0	<i>4.0e+1</i> :10	<i>1.0e-8</i> :10	<i>1.0e-8</i> :10	<i>1.0e-8</i> :10	15/15
MATSUMOTO	<b>1.6</b> (0.8)	<b>1.2</b> (0.6)	<b>1.9</b> (0.3) <sup>*4</sup>	<b>1.9</b> (0.3) <sup>*4</sup>	<b>1.9</b> (0.3) <sup>*4</sup>	15/15
R-DE-10e2-	<b>2.4</b> (2)	<b>2.7</b> (1)	$\infty$	$\infty$	$\infty$ 500	0/15
R-DE-10e5-	<b>2.3</b> (3)	<b>2.4</b> (1)	<b>184</b> (115)	<b>184</b> (44)	<b>184</b> (93)	15/15
RL-SHADE-1	<b>2.0</b> (2)	3.1(3)	372(325)	372(187)	372(538)	2/15
RL-SHADE-1	3.4(3)	4.4(5)	613(20)	613(21)	613(27)	15/15
R-SHADE-10	3.9(5)	3.9(2)	$\infty$	$\infty$	$\infty$ 500	0/15
R-SHADE-10	<b>2.6</b> (4)	5.6(3)	219(30)	219(20)	219(15)	15/15
SOO-Derbel	3.6(0.1)	<b>1.4</b> (0.1)	1054(0.1)	1054(0.1)	1054(0.1)	15/15

Table 7: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_6$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f6</b>	<i>1.0e+5</i> :3.0	<i>2.5e+4</i> :8.4	<i>1.0e+2</i> :16	<i>2.5e+1</i> :54	<i>2.5e-1</i> :254	15/15
MATSUMOTO	<b>1.3</b> <sup>(0.5)</sup>	<b>0.90</b> <sup>(0.7)</sup>	<b>1.7</b> <sup>(1)</sup>	12 <sup>(14)</sup>	$\infty$ 250	0/15
R-DE-10e2-	<b>1.9</b> <sup>(2)</sup>	<b>2.0</b> <sup>(2)</sup>	3.5 <sup>(4)</sup>	<b>2.0</b> <sup>(2)</sup>	<b>6.9</b> <sup>(10)</sup>	4/15
R-DE-10e5-	<b>2.7</b> <sup>(2)</sup>	<b>1.9</b> <sup>(1)</sup>	3.5 <sup>(3)</sup>	<b>2.4</b> <sup>(2)</sup>	16 <sup>(40)</sup>	15/15
RL-SHADE-1	3.0 <sup>(3)</sup>	<b>1.9</b> <sup>(5)</sup>	3.8 <sup>(3)</sup>	<b>2.4</b> <sup>(2)</sup>	14 <sup>(10)</sup>	2/15
RL-SHADE-1	<b>1.9</b> <sup>(3)</sup>	<b>2.6</b> <sup>(3)</sup>	11 <sup>(10)</sup>	6.7 <sup>(5)</sup>	13 <sup>(2)</sup>	15/15
R-SHADE-10	<b>1.9</b> <sup>(2)</sup>	<b>1.6</b> <sup>(1)</sup>	3.3 <sup>(4)</sup>	<b>2.3</b> <sup>(2)</sup>	29 <sup>(35)</sup>	1/15
R-SHADE-10	<b>2.4</b> <sup>(1)</sup>	<b>2.5</b> <sup>(4)</sup>	4.2 <sup>(4)</sup>	<b>2.7</b> <sup>(2)</sup>	<b>2.7</b> <sup>(0.9)</sup>	15/15
SOO-Derbel	<b>1.7</b> <sup>(4)</sup>	<b>1.2</b> <sup>(2)</sup>	<b>1.8</b> <sup>(2)</sup>	<b>2.0</b> <sup>(2)</sup>	6105 <sup>(3636)</sup>	4/15

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Table 8: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_7$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><math>f_7</math></b>	<i>1.6e+2</i> :4.2	<i>1.0e+2</i> :6.2	<i>2.5e+1</i> :20	<i>4.0e+0</i> :54	<i>1.0e+0</i> :324	15/15
MATSUMOTO	<b>1.3</b> (2)	<b>1.8</b> (1)	<b>1.5</b> (0.7)	7.6(9)	5.4(4)	2/15
R-DE-10e2-	<b>2.3</b> (2)	<b>2.3</b> (5)	<b>2.5</b> (1)	5.4(5)	5.3(5)	4/15
R-DE-10e5-	<b>2.0</b> (0.9)	<b>2.1</b> (1)	<b>2.8</b> (1)	4.0(3)	<b>2.7</b> (1.0)	15/15
RL-SHADE-1	<b>1.5</b> (2)	<b>2.3</b> (4)	3.5(2)	5.9(5)	<b>2.8</b> (1)	7/15
RL-SHADE-1	<b>2.2</b> (0.9)	<b>2.1</b> (1)	8.2(9)	12(7)	5.5(2)	15/15
R-SHADE-10	<b>2.9</b> (3)	3.1(2)	4.0(3)	<b>3.9</b> (2)	<b>1.9</b> (3)	10/15
R-SHADE-10	<b>2.3</b> (2)	<b>2.6</b> (5)	<b>2.4</b> (2)	<b>2.6</b> (2)	<b>1.3</b> (2)	15/15
SOO-Derbel	<b>1.1</b> (0.9)	<b>2.2</b> (1)	<b>2.1</b> (2)	4.1(4)	<b>2.1</b> (2)	15/15

Table 9: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_8$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f8</b>	<i>1.0e+4</i> :4.6	<i>6.3e+3</i> :6.8	<i>1.0e+3</i> :18	<i>6.3e+1</i> :54	<i>1.6e+0</i> :258	15/15
MATSUMOTO	<b>1.7</b> (1)	<b>1.9</b> (1)	<b>1.4</b> (0.6)	<b>2.6</b> (2)	$\infty$ 250	0/15
R-DE-10e2-	3.3(4)	<b>2.9</b> (2)	<b>2.9</b> (2)	<b>2.4</b> (1)	<b>9.0</b> (19)	3/15
R-DE-10e5-	<b>2.1</b> (3)	<b>2.0</b> (1)	<b>2.0</b> (1)	<b>2.6</b> (0.6)	22(57)	15/15
RL-SHADE-1	3.3(2)	3.8(4)	3.9(0.5)	3.6(5)	14(12)	2/15
RL-SHADE-1	<b>2.5</b> (2)	3.5(7)	4.9(4)	13(6)	17(3)	15/15
R-SHADE-10	<b>2.4</b> (3)	<b>1.8</b> (2)	<b>2.5</b> (1)	3.4(2)	$\infty$ 500	0/15
R-SHADE-10	3.6(5)	3.0(3)	3.2(4)	3.9(3)	<b>4.3</b> (2)	15/15
SOO-Derbel	<b>1.4</b> (0.5)	<b>1.3</b> (2)	<b>1.4</b> (1.0)	<b>2.1</b> (1)	16(42)	15/15

Table 10: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_9$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f</i><sub>9</sub></b>	<i>2.5e+1:20</i>	<i>1.6e+1:26</i>	<i>1.0e+1:35</i>	<i>4.0e+0:62</i>	<i>1.6e-2:256</i>	15/15
MATSUMOTO	18(20)	34(13)	35(44)	64(65)	$\infty$ <i>250</i>	0/15
R-DE-10e2-	<b>11</b> (7)	10(5)	<b>8.1</b> (3)	11(13)	$\infty$ <i>500</i>	0/15
R-DE-10e5-	12(4)	13(12)	11(6)	10(5)	487(633)	15/15
RL-SHADE-1	13(10)	11(3)	10(4)	15(17)	$\infty$ <i>500</i>	0/15
RL-SHADE-1	54(20)	51(10)	46(11)	45(4)	<b>28</b> (3)	15/15
R-SHADE-10	14(7)	13(8)	10(4)	28(20)	$\infty$ <i>500</i>	0/15
R-SHADE-10	11(4)	<b>10</b> (1)	8.1(3)	<b>8.5</b> (1)	<b>9.1</b> (8)	15/15
SOO-Derbel	<b>6.5</b> (2)	<b>6.3</b> (2)	<b>5.7</b> (1)	<b>5.1</b> (2)	202(348)	15/15

Table 11: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{10}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f10</b>	<i>2.5e+6:2.9</i>	<i>6.3e+5:7.0</i>	<i>2.5e+5:17</i>	<i>6.3e+3:54</i>	<i>2.5e+1:297</i>	15/15
MATSUMOTO	<b>1.5</b> (2)	<b>1.9</b> (1)	<b>1.4</b> (0.9)	6.4(9)	$\infty$ <i>250</i>	0/15
R-DE-10e2-	<b>1.4</b> (2)	<b>1.2</b> (0.7)	<b>1.2</b> (1)	<b>2.6</b> (0.7)	$\infty$ <i>500</i>	0/15
R-DE-10e5-	<b>1.3</b> (3)	<b>1.9</b> (2)	<b>1.2</b> (0.4)	7.1(4)	169(117)	15/15
RL-SHADE-1	<b>1.6</b> (3)	<b>1.6</b> (0.9)	<b>1.5</b> (2)	4.5(3)	<b>12</b> (15)	2/15
RL-SHADE-1	<b>1.6</b> (1)	<b>2.3</b> (2)	<b>1.4</b> (0.5)	14(6)	15(1)	15/15
R-SHADE-10	<b>1.7</b> (1)	<b>1.7</b> (3)	<b>1.2</b> (2)	6.7(8)	$\infty$ <i>500</i>	0/15
R-SHADE-10	<b>1.8</b> (3)	<b>1.2</b> (1)	<b>1.3</b> (2)	<b>3.5</b> (1)	<b>2.2</b> (0.6)	15/15
SOO-Derbel	<b>1.4</b> (0.7)	<b>0.74</b> (0.8)	<b>0.56</b> (0.6)	4.6(2)	74(254)	15/15

Table 12: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{11}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f11</b>	<i>1.0e+6:3.0</i>	<i>6.3e+4:6.2</i>	<i>6.3e+2:16</i>	<i>6.3e+1:74</i>	<i>6.3e-1:298</i>	15/15
MATSUMOTO	<b>1.4</b> (2)	<b>2.5</b> (2)	4.7(1)	8.9(6)	$\infty$ 250	0/15
R-DE-10e2-	<b>2.3</b> (4)	<b>2.0</b> (2)	5.1(5)	3.8(3)	$\infty$ 500	0/15
R-DE-10e5-	<b>1.9</b> (2)	3.3(1)	4.6(2)	36(121)	4079(4104)	5/15
RL-SHADE-1	<b>1.5</b> (0.8)	<b>2.2</b> (5)	<b>3.7</b> (2)	<b>3.1</b> (3)	$\infty$ 500	0/15
RL-SHADE-1	<b>3.0</b> (6)	3.4(5)	6.6(5)	6.7(3)	<b>15</b> (3)	15/15
R-SHADE-10	<b>2.0</b> (3)	<b>1.8</b> (2)	4.0(5)	4.0(4)	$\infty$ 500	0/15
R-SHADE-10	<b>1.2</b> (1)	4.2(3)	<b>3.9</b> (3)	<b>2.2</b> (0.9)	<b>2.7</b> (1)	15/15
SOO-Derbel	<b>0.98</b> (0.3)	<b>1.6</b> (3)	5.0(2)	7.5(20)	2683(2985)	6/15

Table 13: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{12}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f12</b>	<i>4.0e+7:3.6</i>	<i>1.6e+7:7.6</i>	<i>4.0e+6:19</i>	<i>1.6e+4:52</i>	<i>1.0e+0:268</i>	15/15
MATSUMOTO	<b>1.2</b> <sup>(0.6)</sup>	<b>1.4</b> <sup>(1.0)</sup>	<b>1.7</b> <sup>(0.5)</sup>	<b>3.2</b> <sup>(1)</sup>	$\infty$ <i>250</i>	0/15
R-DE-10e2-	<b>1.8</b> <sup>(2)</sup>	<b>2.6</b> <sup>(4)</sup>	3.1 <sup>(1)</sup>	<b>4.5</b> <sup>(2)</sup>	$\infty$ <i>500</i>	0/15
R-DE-10e5-	<b>1.6</b> <sup>(2)</sup>	<b>2.3</b> <sup>(2)</sup>	<b>2.4</b> <sup>(2)</sup>	5.3 <sup>(3)</sup>	112 <sup>(66)</sup>	15/15
RL-SHADE-1	<b>1.4</b> <sup>(1)</sup>	<b>2.5</b> <sup>(1.0)</sup>	3.3 <sup>(1)</sup>	4.8 <sup>(0.9)</sup>	$\infty$ <i>500</i>	0/15
RL-SHADE-1	<b>2.1</b> <sup>(2)</sup>	4.0 <sup>(1)</sup>	6.9 <sup>(10)</sup>	39 <sup>(11)</sup>	29 <sup>(4)</sup>	15/15
R-SHADE-10	<b>2.1</b> <sup>(0.6)</sup>	<b>2.7</b> <sup>(1)</sup>	<b>2.9</b> <sup>(1)</sup>	5.6 <sup>(2)</sup>	$\infty$ <i>500</i>	0/15
R-SHADE-10	<b>0.96</b> <sup>(0.8)</sup>	<b>1.8</b> <sup>(1)</sup>	<b>2.3</b> <sup>(2)</sup>	6.0 <sup>(2)</sup>	<b>7.1</b> <sup>(8)</sup>	15/15
SOO-Derbel	<b>0.50</b> <sup>(0.1)</sup>	<b>0.66</b> <sup>(2)</sup>	<b>1.2</b> <sup>(0.7)</sup>	5.4 <sup>(2)</sup>	<b>6.8</b> <sup>(1)</sup>	15/15

Table 14: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{13}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f13</b>	<i>1.0e+3:2.8</i>	<i>6.3e+2:8.4</i>	<i>4.0e+2:17</i>	<i>6.3e+1:52</i>	<i>6.3e-2:264</i>	15/15
MATSUMOTO	<b>2.0</b> <sub>(1.0)</sub>	<b>1.8</b> <sub>(1)</sub>	<b>1.5</b> <sub>(0.5)</sub>	<b>1.7</b> <sub>(0.3)</sub> <sup>*2</sup>	$\infty$ <i>250</i>	0/15
R-DE-10e2-	<b>2.5</b> <sub>(2)</sub>	<b>2.7</b> <sub>(0.7)</sub>	<b>2.4</b> <sub>(2)</sub>	4.5 <sub>(1)</sub>	$\infty$ <i>500</i>	0/15
R-DE-10e5-	<b>2.0</b> <sub>(2)</sub>	<b>3.0</b> <sub>(2)</sub>	3.3 <sub>(2)</sub>	4.6 <sub>(3)</sub>	739 <sub>(735)</sub>	13/15
RL-SHADE-1	<b>1.7</b> <sub>(0.8)</sub>	<b>2.6</b> <sub>(3)</sub>	3.4 <sub>(2)</sub>	3.9 <sub>(0.9)</sub>	$\infty$ <i>500</i>	0/15
RL-SHADE-1	3.4 <sub>(8)</sub>	5.0 <sub>(5)</sub>	6.7 <sub>(5)</sub>	23 <sub>(6)</sub>	<b>28</b> <sub>(2)</sub>	15/15
R-SHADE-10	<b>2.5</b> <sub>(5)</sub>	3.4 <sub>(2)</sub>	<b>2.8</b> <sub>(1)</sub>	5.3 <sub>(5)</sub>	$\infty$ <i>500</i>	0/15
R-SHADE-10	<b>1.2</b> <sub>(0.8)</sub>	<b>1.9</b> <sub>(5)</sub>	<b>2.8</b> <sub>(3)</sub>	4.2 <sub>(2)</sub>	<b>4.8</b> <sub>(2)</sub>	15/15
SOO-Derbel	<b>1.0</b> <sub>(2)</sub>	<b>0.96</b> <sub>(0.7)</sub>	<b>1.2</b> <sub>(0.4)</sub>	<b>3.9</b> <sub>(0.3)</sub>	29 <sub>(26)</sub>	15/15

Table 15: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{14}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f14</b>	<i>1.6e+1:3.0</i>	<i>1.0e+1:10</i>	<i>6.3e+0:15</i>	<i>2.5e-1:53</i>	<i>1.0e-5:251</i>	15/15
MATSUMOTO	<b>2.1</b> (1)	<b>1.4</b> (0.8)	<b>1.3</b> (0.5)	<b>2.3</b> (1)	$\infty$ 250	0/15
R-DE-10e2-	3.4(6)	<b>1.7</b> (2)	<b>2.3</b> (2)	<b>3.4</b> (2)	$\infty$ 500	0/15
R-DE-10e5-	3.5(2)	<b>2.1</b> (2)	<b>1.8</b> (2)	3.7(2)	579(1050)	14/15
RL-SHADE-1	<b>2.3</b> (2)	<b>2.2</b> (3)	<b>2.6</b> (3)	3.6(1)	$\infty$ 500	0/15
RL-SHADE-1	5.3(3)	3.1(4)	3.5(3)	20(7)	<b>29</b> (2)	15/15
R-SHADE-10	<b>2.7</b> (4)	<b>1.6</b> (2)	<b>2.1</b> (2)	3.9(0.5)	$\infty$ 500	0/15
R-SHADE-10	3.2(2)	<b>1.3</b> (0.7)	<b>1.4</b> (1)	4.4(0.6)	<b>5.3</b> (0.9)	15/15
SOO-Derbel	<b>1.0</b> (1)	<b>0.59</b> (0.8)	<b>0.74</b> (0.9)	3.6(1)	1342(1364)	12/15



Table 16: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{15}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f15</b>	<i>1.6e+2:3.0</i>	<i>1.0e+2:13</i>	<i>6.3e+1:24</i>	<i>4.0e+1:55</i>	<i>1.6e+1:289</i>	5/5
MATSUMOTO	<b>2.4</b> (2)	<b>1.3</b> (0.1)	<b>1.6</b> (2)	<b>1.7</b> (1.0)	<b>0.97</b> (0.8)	10/15
R-DE-10e2-	<b>2.6</b> (2)	<b>1.6</b> (0.8)	<b>1.8</b> (0.8)	<b>1.9</b> (2)	<b>2.1</b> (0.8)	10/15
R-DE-10e5-	<b>2.9</b> (2)	<b>1.6</b> (1)	<b>1.8</b> (1)	<b>1.6</b> (0.4)	3.4(7)	15/15
RL-SHADE-1	<b>2.5</b> (2)	<b>2.8</b> (2)	3.1(0.8)	<b>2.0</b> (0.4)	3.0(3)	7/15
RL-SHADE-1	<b>1.9</b> (3)	<b>2.6</b> (4)	6.1(5)	7.3(5)	7.6(3)	15/15
R-SHADE-10	4.8(5)	<b>2.3</b> (2)	<b>2.4</b> (1)	<b>2.0</b> (0.4)	<b>2.8</b> (3)	8/15
R-SHADE-10	4.2(6)	<b>2.0</b> (1)	<b>2.4</b> (1)	<b>2.4</b> (3)	<b>2.0</b> (0.6)	15/15
SOO-Derbel	<b>1.1</b> (2)	<b>0.64</b> (0.5)	<b>1.3</b> (0.7)	<b>1.8</b> (2)	<b>1.6</b> (0.6)	15/15

Table 17: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{16}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f16</b>	<i>4.0e+1:4.8</i>	<i>2.5e+1:16</i>	<i>1.6e+1:46</i>	<i>1.0e+1:120</i>	<i>4.0e+0:334</i>	15/15
MATSUMOTO	<b>1.4</b> (2)	<b>0.86</b> (0.7)	<b>1.2</b> (2)	<b>1.2</b> (0.9)	3.9(4)	3/15
R-DE-10e2-	<b>1.4</b> (0.8)	<b>1.9</b> (2)	<b>1.1</b> (1)	<b>0.95</b> (1)	5.3(4)	4/15
R-DE-10e5-	<b>1.0</b> (0.9)	<b>1.2</b> (1.0)	<b>1.7</b> (2)	<b>1.7</b> (2)	4.5(5)	15/15
RL-SHADE-1	<b>1.8</b> (0.8)	<b>1.4</b> (2)	<b>1.3</b> (0.5)	<b>1.2</b> (0.9)	<b>1.7</b> (1.0)	10/15
RL-SHADE-1	<b>1.1</b> (1)	<b>1.3</b> (0.6)	<b>1.6</b> (3)	<b>1.6</b> (2)	7.5(5)	15/15
R-SHADE-10	<b>1.9</b> (2)	<b>1.6</b> (2)	<b>1.6</b> (1)	<b>1.4</b> (0.7)	<b>2.2</b> (4)	8/15
R-SHADE-10	<b>1.7</b> (2)	<b>0.85</b> (0.5)	<b>1.5</b> (2)	<b>1.1</b> (0.8)	<b>2.7</b> (1)	15/15
SOO-Derbel	<b>1.7</b> (2)	<b>1.6</b> (1)	<b>1.9</b> (1)	<b>1.2</b> (0.7)	<b>0.85</b> (0.5)	15/15

Table 18: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{17}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f17</i></b>	<i>1.0e+1:5.2</i>	<i>6.3e+0:26</i>	<i>4.0e+0:57</i>	<i>2.5e+0:110</i>	<i>6.3e-1:412</i>	15/15
MATSUMOTO	3.1(2)	<b>1.2</b> (1)	<b>1.1</b> (0.5)	<b>1.9</b> (1)	9.2(8)	1/15
R-DE-10e2-	4.1(4)	<b>1.8</b> (1)	<b>1.5</b> (0.8)	<b>1.2</b> (0.7)	<b>0.94</b> (0.2)	12/15
R-DE-10e5-	4.2(2)	<b>1.8</b> (2)	<b>1.7</b> (1)	5.0(7)	7.4(11)	15/15
RL-SHADE-1	<b>2.4</b> (2)	<b>1.0</b> (1)	<b>1.1</b> (0.5)	<b>1.3</b> (0.8)	<b>1.8</b> (2)	8/15
RL-SHADE-1	<b>2.8</b> (2)	<b>1.7</b> (1)	3.1(3)	4.3(2)	4.6(1)	15/15
R-SHADE-10	3.3(4)	<b>2.1</b> (2)	<b>1.9</b> (1)	<b>1.8</b> (2)	<b>2.7</b> (2)	6/15
R-SHADE-10	<b>2.8</b> (2)	<b>1.3</b> (0.4)	<b>1.2</b> (0.9)	<b>1.3</b> (1)	<b>1.0</b> (0.4)	15/15
SOO-Derbel	<b>1.4</b> (2)	<b>0.64</b> (0.6)	<b>0.86</b> (0.7)	<b>0.93</b> (0.5)	<b>0.92</b> (0.6)	15/15

Table 19: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{18}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f18</b>	<i>6.3e+1:3.4</i>	<i>4.0e+1:7.2</i>	<i>2.5e+1:20</i>	<i>1.6e+1:58</i>	<i>1.6e+0:318</i>	15/15
MATSUMOTO	<b>1.9</b> (2)	<b>1.4</b> (2)	<b>1.1</b> (0.9)	<b>0.85</b> (1)	$\infty$ 250	0/15
R-DE-10e2-	<b>2.1</b> (2)	<b>2.5</b> (2)	<b>1.9</b> (2)	<b>1.3</b> (0.3)	4.2(2)	5/15
R-DE-10e5-	<b>2.6</b> (1)	<b>2.7</b> (6)	<b>2.1</b> (2)	<b>1.3</b> (0.5)	<b>2.9</b> (3)	15/15
RL-SHADE-1	<b>1.5</b> (3)	<b>1.7</b> (1)	<b>2.2</b> (2)	<b>1.4</b> (1)	4.4(3)	5/15
RL-SHADE-1	<b>1.1</b> (0.8)	<b>2.1</b> (1)	3.1(2)	3.9(3)	8.2(3)	15/15
R-SHADE-10	<b>1.5</b> (1)	<b>2.8</b> (5)	<b>1.8</b> (2)	<b>1.7</b> (0.8)	12(13)	2/15
R-SHADE-10	<b>1.4</b> (1)	<b>1.5</b> (2)	<b>1.8</b> (2)	<b>1.5</b> (0.9)	<b>1.5</b> (1)	15/15
SOO-Derbel	<b>0.76</b> (0.8)	<b>1.0</b> (1)	<b>0.76</b> (0.4)	<b>0.79</b> (0.5)	<b>1.6</b> (0.7)	15/15

Table 20: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{19}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f19</b>	<i>1.6e-1:172</i>	<i>1.0e-1:242</i>	<i>6.3e-2:675</i>	<i>4.0e-2:3078</i>	<i>2.5e-2:4946</i>	15/15
MATSUMOTO	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ <i>250</i>	0/15
R-DE-10e2-	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ <i>500</i>	0/15
R-DE-10e5-	1109(1356)	1951(2356)	3319(4999)	1163(1634)	1470(1238)	1/15
RL-SHADE-1	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ <i>500</i>	0/15
RL-SHADE-1	273(149)	269(150)	134(116)	70(52)	116(136)	9/15
R-SHADE-10	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$ <i>500</i>	0/15
R-SHADE-10	<b>90</b> (115)	<b>111</b> (153)	<b>85</b> (75)	<b>68</b> (108)	<b>61</b> (82)	11/15
SOO-Derbel	<b>3.7</b> (1)	<b>10</b> (28)	<b>6.3</b> (0.3)	<b>1.4</b> (0.1)	<b>0.90</b> (3)	15/15

Table 21: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{20}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><math>f_{20}</math></b>	<i>6.3e+3:5.1</i>	<i>4.0e+3:8.4</i>	<i>4.0e+1:15</i>	<i>2.5e+0:69</i>	<i>1.0e+0:851</i>	15/15
MATSUMOTO	<b>1.7</b> (1)	<b>1.4</b> (1)	<b>1.9</b> (1)	4.1(3)	$\infty$ 250	0/15
R-DE-10e2-	<b>2.1</b> (3)	<b>1.8</b> (2)	3.5(3)	<b>2.0</b> (0.7)	<b>1.1</b> (1)	7/15
R-DE-10e5-	<b>2.4</b> (2)	<b>1.8</b> (0.8)	<b>3.3</b> (2)	<b>2.2</b> (0.8)	<b>0.95</b> (1)	15/15
RL-SHADE-1	<b>1.4</b> (1)	<b>1.3</b> (1)	4.3(3)	<b>2.0</b> (0.6)	<b>1.3</b> (1)	6/15
RL-SHADE-1	<b>2.0</b> (2)	<b>1.8</b> (2)	12(4)	11(6)	5.6(2)	15/15
R-SHADE-10	<b>2.5</b> (3)	<b>2.2</b> (3)	4.0(3)	<b>2.5</b> (1)	<b>1.4</b> (2)	6/15
R-SHADE-10	<b>2.3</b> (4)	<b>1.9</b> (2)	3.9(2)	3.2(3)	<b>1.9</b> (0.8)	15/15
SOO-Derbel	<b>0.88</b> (0.1)	<b>1.5</b> (0.1)	11(0.0)	3.7(7e-3)	<b>1.2</b> (6e-4)	15/15

Table 22: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{21}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f</i>21</b>	<i>4.0e+1:3.9</i>	<i>2.5e+1:11</i>	<i>1.6e+1:31</i>	<i>6.3e+0:73</i>	<i>1.6e+0:347</i>	5/5
MATSUMOTO	<b>1.3</b> <sup>(0.6)</sup>	<b>1.3</b> <sup>(2)</sup>	<b>0.73</b> <sup>(0.8)</sup>	<b>1.2</b> <sup>(0.7)</sup>	<b>1.5</b> <sup>(1)</sup>	7/15
R-DE-10e2-	<b>1.8</b> <sup>(1)</sup>	<b>1.2</b> <sup>(0.8)</sup>	<b>0.82</b> <sup>(0.8)</sup>	<b>1.6</b> <sup>(1)</sup>	4.7 <sup>(4)</sup>	4/15
R-DE-10e5-	<b>2.2</b> <sup>(3)</sup>	<b>1.6</b> <sup>(0.7)</sup>	<b>1.1</b> <sup>(0.7)</sup>	3.6 <sup>(6)</sup>	33 <sup>(23)</sup>	15/15
RL-SHADE-1	<b>1</b> <sup>(0.7)</sup>	<b>1.4</b> <sup>(1)</sup>	<b>1.9</b> <sup>(2)</sup>	<b>2.1</b> <sup>(1.0)</sup>	<b>2.3</b> <sup>(3)</sup>	7/15
RL-SHADE-1	<b>1.5</b> <sup>(2)</sup>	<b>1.5</b> <sup>(2)</sup>	<b>1.6</b> <sup>(1)</sup>	5.0 <sup>(3)</sup>	4.5 <sup>(2)</sup>	15/15
R-SHADE-10	<b>1.9</b> <sup>(2)</sup>	<b>1.5</b> <sup>(3)</sup>	<b>1.3</b> <sup>(1)</sup>	<b>2.0</b> <sup>(0.7)</sup>	<b>2.1</b> <sup>(4)</sup>	8/15
R-SHADE-10	<b>2.1</b> <sup>(0.8)</sup>	<b>2.2</b> <sup>(2)</sup>	<b>1.6</b> <sup>(1)</sup>	4.5 <sup>(9)</sup>	5.3 <sup>(10)</sup>	15/15
SOO-Derbel	<b>1.3</b> <sup>(1.0)</sup>	<b>1.1</b> <sup>(2)</sup>	<b>0.88</b> <sup>(0.7)</sup>	<b>0.99</b> <sup>(0.6)</sup>	<b>0.90</b> <sup>(0.8)</sup>	15/15

Table 23: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{22}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b>f22</b>	<i>6.3e+1</i> :3.6	<i>4.0e+1</i> :15	<i>2.5e+1</i> :32	<i>1.0e+1</i> :71	<i>1.6e+0</i> :341	5/5
MATSUMOTO	<b>1.6</b> (1)	<b>1.1</b> (0.9)	<b>1.1</b> (0.6)	<b>1.1</b> (0.7)	<b>2.3</b> (3)	5/15
R-DE-10e2-	<b>2.8</b> (3)	<b>1.5</b> (2)	<b>1.2</b> (0.6)	<b>1.3</b> (0.9)	<b>2.3</b> (3)	7/15
R-DE-10e5-	<b>1.4</b> (0.7)	<b>1.3</b> (1)	<b>1.5</b> (2)	<b>2.1</b> (1)	17(29)	15/15
RL-SHADE-1	<b>1.4</b> (2)	<b>1.9</b> (2)	<b>1.3</b> (0.7)	<b>1.6</b> (0.7)	<b>2.9</b> (4)	6/15
RL-SHADE-1	<b>2.5</b> (3)	<b>2.0</b> (1.0)	<b>1.9</b> (3)	3.7(5)	4.2(2)	15/15
R-SHADE-10	<b>2.7</b> (5)	<b>1.5</b> (1)	<b>1.1</b> (0.4)	<b>1.6</b> (2)	6.4(11)	3/15
R-SHADE-10	3.0(2)	<b>1.8</b> (1)	<b>1.2</b> (1)	<b>1.5</b> (0.8)	3.3(7)	15/15
SOO-Derbel	<b>1.5</b> (2)	<b>0.94</b> (0.6)	<b>0.77</b> (0.7)	<b>1.0</b> (0.5)	<b>1.00</b> (1.0)	15/15



Table 24: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best } 2009}$  on  $f_{23}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best } 2009}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f23</i></b>	<i>1.0e+1:3.0</i>	<i>6.3e+0:9.0</i>	<i>4.0e+0:33</i>	<i>2.5e+0:84</i>	<i>1.0e+0:518</i>	15/15
MATSUMOTO	<b>1.8</b> (2)	<b>1.7</b> (2)	<b>2.4</b> (3)	<b>3.9</b> (6)	$\infty$ <i>250</i>	0/15
R-DE-10e2-	<b>1.6</b> (2)	<b>1.1</b> (1)	<b>2.1</b> (2)	4.5(3)	14(10)	1/15
R-DE-10e5-	<b>2.0</b> (1)	<b>2.4</b> (3)	3.8(4)	6.0(7)	34(34)	15/15
RL-SHADE-1	<b>2.8</b> (3)	<b>2.8</b> (1)	3.1(3)	7.3(9)	$\infty$ <i>500</i>	0/15
RL-SHADE-1	<b>2.2</b> (2)	<b>2.3</b> (0.9)	<b>2.5</b> (1)	5.1(6)	18(9)	15/15
R-SHADE-10	3.1(4)	<b>2.6</b> (1)	3.5(4)	5.2(6)	$\infty$ <i>500</i>	0/15
R-SHADE-10	<b>2.9</b> (3)	<b>2.1</b> (2)	3.2(3)	6.3(7)	<b>6.2</b> (3)	15/15
SOO-Derbel	<b>1.7</b> (2)	<b>2.9</b> (5)	<b>2.5</b> (3)	<b>3.8</b> (2)	<b>1.4</b> (0.2)	15/15

Table 25: 05-D, running time excess  $\text{ERT}/\text{ERT}_{\text{best 2009}}$  on  $f_{24}$  for given run-length based budgets (0.5D, 1.2D, 3D, 10D, and 50D function evaluations). The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding  $\text{ERT}_{\text{best 2009}}$  (preceded by the target  $\Delta f$ -value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries with succeeding star are statistically significantly better (according to the rank-sum test) compared to all other algorithms in the table, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k$  following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
<b><i>f</i>24</b>	<i>6.3e+1:15</i>	<i>4.0e+1:37</i>	<i>4.0e+1:37</i>	<i>2.5e+1:118</i>	<i>1.6e+1:692</i>	15/15
MATSUMOTO	<b>1.9</b> (3)	3.1(2)	3.1(4)	10(8)	5.3(4)	1/15
R-DE-10e2-	<b>1.7</b> (0.9)	<b>2.7</b> (1)	<b>2.7</b> (1)	3.9(4)	11(18)	1/15
R-DE-10e5-	<b>1.9</b> (2)	<b>2.0</b> (2)	<b>2.0</b> (2)	<b>2.0</b> (3)	<b>2.0</b> (1)	15/15
RL-SHADE-1	<b>2.4</b> (3)	3.7(1)	3.7(2)	5.3(4)	11(14)	1/15
RL-SHADE-1	<b>2.8</b> (3)	8.0(6)	8.0(7)	10(4)	5.0(3)	15/15
R-SHADE-10	<b>2.8</b> (1)	3.3(2)	3.3(2)	5.7(4)	11(16)	1/15
R-SHADE-10	<b>2.4</b> (3)	3.1(2)	3.1(2)	<b>3.0</b> (2)	<b>1.8</b> (1)	15/15
SOO-Derbel	<b>1.1</b> (2)	5.1(4)	5.1(3)	5.2(2)	<b>1.8</b> (1)	15/15

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